2.1 DEFINING AVERAGE AND INSTANTANEOUS RATES OF CHANGE AT A POINT

1. The average rate of change over an interval becomes a better and better estimate as we ___narrow___ the interval, until eventually, when we narrow the interval to 0, the ___average___ rate of change will equal the ___instantaneous___ rate of change.

2. Give an equivalent expression for the difference quotient.

$$\frac{f(x)-f(a)}{x-a}=\frac{f(a+h)-f(a)}{h}$$

2.2 DEFINING THE DERIVATIVE OF A FUNCTION AND USING DERIVATIVE NOTATION

1. Give other expressions for the derivative.

$$y'(x)=y'=f'(x)=\frac{dy}{dx}$$

2. Write an expression for instantaneous rate of change at $x=a$.

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \text{ or } f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

3. The ___derivative___ at a point is equal to the ___slope___ of the ___tangent___ line at that point.

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2.3 ESTIMATING DERIVATIVES OF A FUNCTION AT A POINT

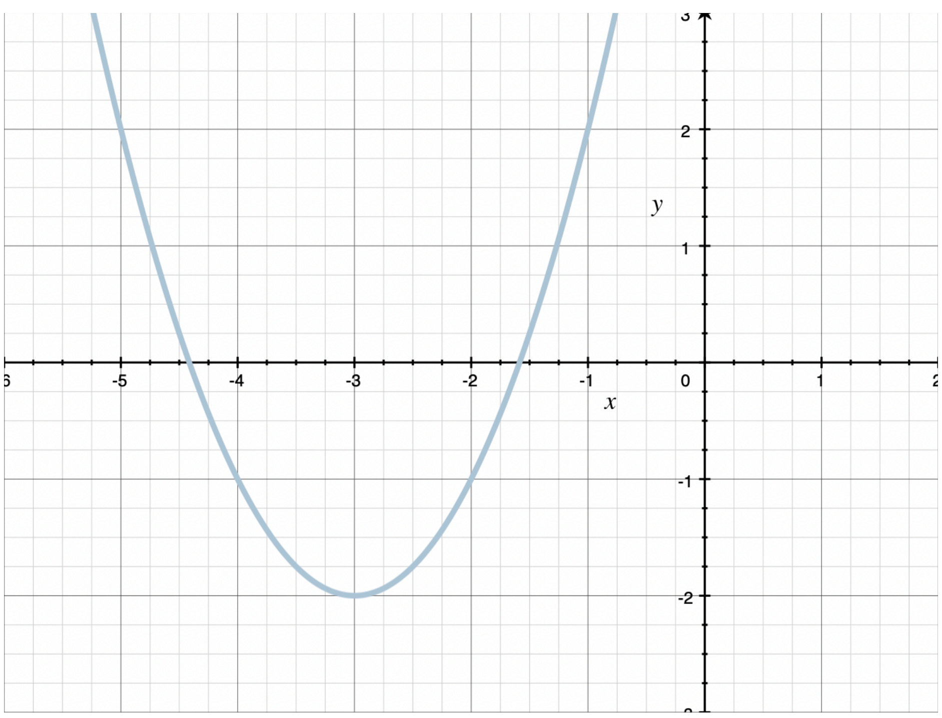
1. Use the table to estimate the derivative of $f(x)$ at $x = 3.5$.

x	1	2	3	4	5
f(x)	-3	4	2	5	-1

Use the difference quotient.

$$f'(3.5) \approx \frac{5-2}{4-3} \approx \frac{3}{1} \approx 3$$

2. Use values near $x = -2$ from the graph to estimate the derivative of the function at $x = -2$.



The graph passes through $(-3, -2)$ and $(-1, 2)$, so plug those points into the difference quotient.

$$f'(-2) \approx \frac{-2-(2)}{-3-(-1)} \approx \frac{-2-2}{-3+1} \approx \frac{-4}{-2} \approx 2$$

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2.4 CONNECTING DIFFERENTIABILITY AND CONTINUITY: DETERMINING WHEN DERIVATIVES DO AND DO NOT EXIST

1. If a function is differentiable at a point, then it's continuous there, but even if a function is continuous at a point, it's not necessarily differentiable there.

2. Name two conditions for differentiability.

    1. Continuity

    2. Smoothness

2.5 APPLYING THE POWER RULE

1. Use power rule to give the derivative of each power function.

$\frac{d}{dx}(-6x) = -6$ $\frac{d}{dx}(4x^2) = 8x$ $\frac{d}{dx}(3x^0) = 0$ $\frac{d}{dx}(-\pi x^{-3}) = 3\pi x^{-4}$

2.6 DERIVATIVE RULES: CONSTANT, SUM, DIFFERENCE, AND CONSTANT MULTIPLE

1. Find the derivative of each constant.

$\frac{d}{dx}(3) = 0$ $\frac{d}{dx}(-7) = 0$ $\frac{d}{dx}(\pi^2) = 0$ $\frac{d}{dx}(3e) = 0$

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2. Find the derivative of each polynomial.

$$\frac{d}{dx}(3x^2 - 6x + 4) = 6x - 6$$

$$\frac{d}{dx}(8\pi^4 - 2x^3 + 6x^{-1}) = -6x^2 - 6x^{-2}$$

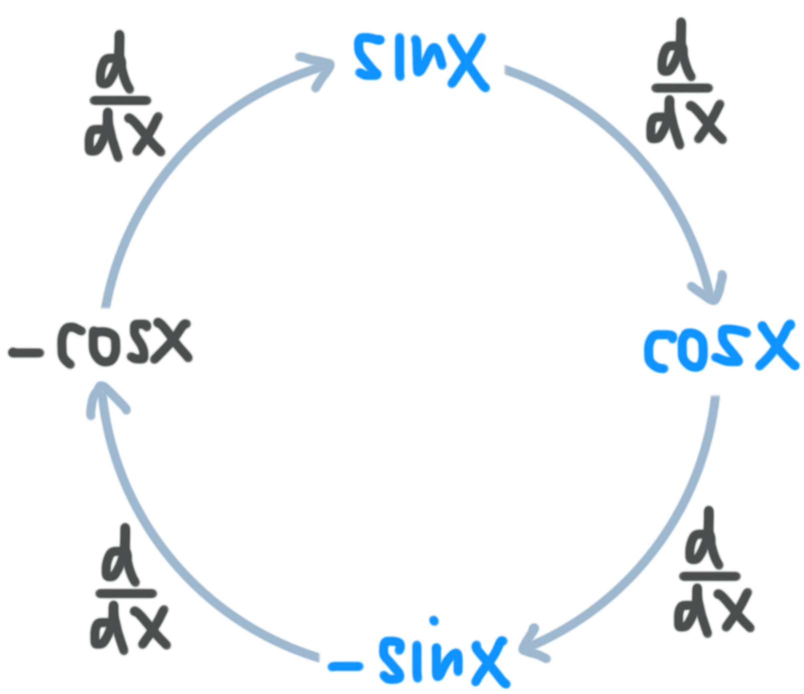
2.7 DERIVATIVES OF $\cos x$, $\sin x$, e^x, AND $\ln x$

1. Complete the table with the derivative of each function.

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\ln x) = \frac{1}{x}$$

2. Complete the circle diagram with each derivative.



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2.8 THE PRODUCT RULE

1. Use product rule to differentiate each function.

$f(x) = g(x)h(x)$ $\quad$ $f'(x) = g'(x)h(x) + g(x)h'(x)$

$f(x) = e^x \sin x$ $\quad$ $f'(x) = e^x \sin x + e^x \cos x$

$f(x) = x^{-3} \ln x$ $\quad$ $f'(x) = \dfrac{1 - 3\ln x}{x^4}$

2. If the table gives values of the differentiable functions f and g and their derivatives at selected values of x, and if h is the function defined by $h(x) = -3g(x) + f(x)g(x)$, find $h'(2)$.

x	f(x)	f'(x)	g(x)	g'(x)
1	1	4	2	-2
2	-3	1	1	-3

$h'(x) = -3g'(x) + f'(x)g(x) + f(x)g'(x)$

$h'(2) = -3g'(2) + f'(2)g(2) + f(2)g'(2)$

$h'(2) = -3(-3) + (1)(1) + (-3)(-3)$

$h'(2) = 9 + 1 + 9$

$h'(2) = 19$

2.9 THE QUOTIENT RULE

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1. If $s(t) = x(t)/r(t)$, what is $s'(t)$?

$$s'(t) = \frac{x'(t)r(t) - x(t)r'(t)}{(r(t))^2}$$

2. Use quotient rule to find the derivative of each rational function.

$$f(x) = \frac{\sin x - e^x}{4x^2}$$

$$f'(x) = \frac{(\cos x - e^x)(4x^2) - (\sin x - e^x)(8x)}{(4x^2)^2}$$

$$f'(x) = \frac{4x^2(\cos x - e^x) - 8x(\sin x - e^x)}{16x^4}$$

$$g(x) = \frac{9x \ln x}{x+1}$$

$$g'(x) = \frac{\left[(9)(\ln x) + (9x)\left(\frac{1}{x}\right)\right](x+1) - (9x \ln x)(1)}{(x+1)^2}$$

$$g'(x) = \frac{(9 \ln x + 9)(x+1) - 9x \ln x}{(x+1)^2}$$

2.10 FINDING THE DERIVATIVES OF TANGENT, COTANGENT, SECANT, AND/OR COSECANT FUNCTIONS

1. Re-order the derivatives, below, to line up with the corresponding trig functions, above.

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$\sin x$	$\cos x$	$\tan x$	$\csc x$	$\sec x$	$\cot x$
$\cos x$	$-\sin x$	$\sec^2 x$	$-\csc x \cot x$	$\sec x \tan x$	$-\csc^2 x$

2. Use trig identities to prove the derivative of $\cot x$.

$$f(x) = \cot x = \frac{\cos x}{\sin x}$$

$$f'(x) = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\frac{1}{\sin x} \cdot \frac{1}{\sin x}$$

$$= -\csc x \cdot \csc x = -\csc^2 x$$

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