4.1 INTERPRETING THE MEANING OF THE DERIVATIVE IN CONTEXT

1. A population is modeled by $p(t) = 1.21(1.0144)^t$, where t is the number of years since 2010. Match the notation on the left with the corresponding statement on the right.

$p(5)$	The number of people by which the population is changing in the year 2015
$p'(5)$	The rate at which the number of people in the population is changing in the year 2015
$p''(5)$	The total population in 2015

2. Every day, Kaylin records her weight. She models the data with $w(t) = -2.5t^4 - 0.1x^2 + 0.2x + 146$, where t is the number of days since she started recording her weight. Explain the meaning of each statement.

a. $w'(7) < 0$: ____

b. $w'(7) > 0$: ____

c. $w'(t) < 0$ for $t < 7$, $w'(t) > 0$ for $t > 7$ and $w'(7) = 0$:

____

3. A drone moves along the curve $h(t) = \frac{1}{3}t^3 - 2t^2 + 3t$, where $h(t)$ gives the number of feet traveled after t seconds. Complete the table.

t	0	
h(t)		0
h'(t)		0
h''(t)		

4.2 STRAIGHT-LINE MOTION: CONNECTING POSITION, VELOCITY, AND ACCELERATION

1. Velocity is the first derivative of ________________, and acceleration is its second derivative. So acceleration is the first derivative of ________________. ________________ is the absolute value of velocity.

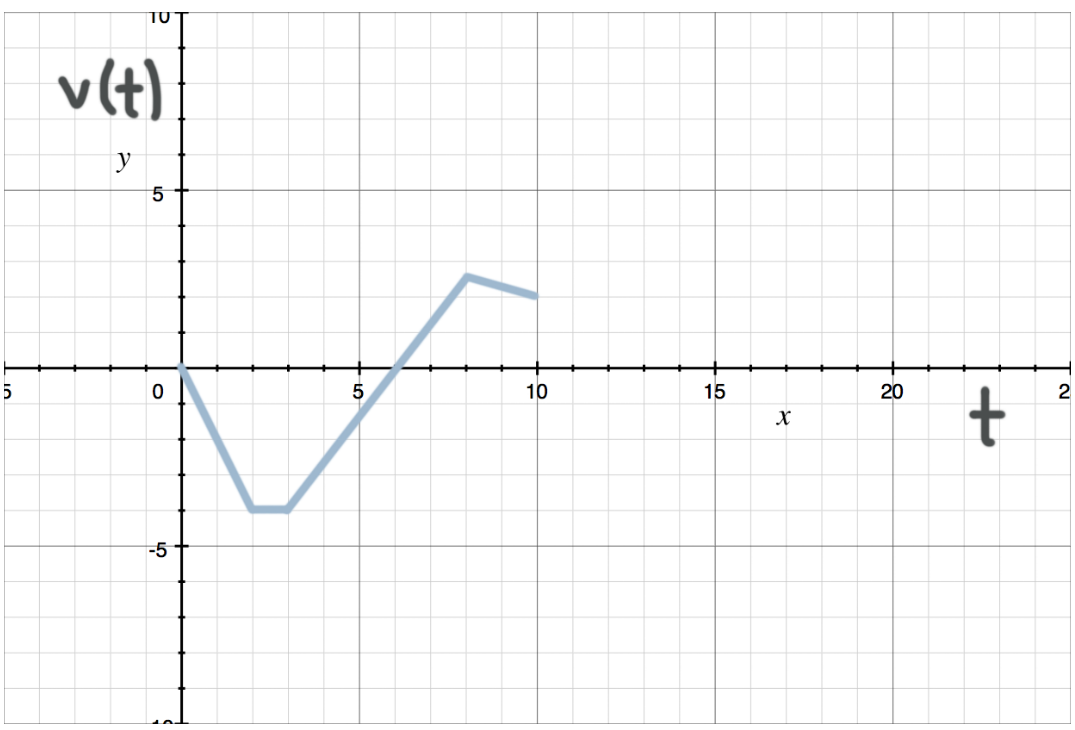
2. Hannah rides a bike from left to right along a straight road. Say whether each statement is true or false, and explain why.

If Hannah is moving to the left, her velocity is positive.	
If Hannah is moving to the left and her velocity is decreasing, Hannah is speeding up.	
If Hannah's velocity changes from negative to positive or from positive to negative, it means she's changed direction.	
If Hannah biked 20 miles and finished her ride where she started, her displacement is 20 miles.	

3. A train's position on an east-west track is modeled by $x(t) = t^3 - 3t^2 - 10$, where time t is measured in minutes and position $x(t)$ is measured in miles. Complete the table with the units of each value in miles, miles/minute, or miles/minute^2.

Position	
Displacement	
Total distance	
Velocity	
Acceleration	
Average velocity	
Average acceleration	
Speed	

4. The graph shows velocity $v(t)$, measured in feet per second, of a squirrel running across a telephone line. Say in the table where velocity and acceleration are positive, negative, or zero, and whether the squirrel is speeding up, slowing down, or neither.



Time (seconds)	Velocity (ft/sec)	Acceleration (ft/sec^2)	Speeding up, slowing down, or neither
(0,2)			
(2,3)			
(3,6)			
(6,8)			
(8,10)			

5. The table shows information about a marble that's rolling back and forth, where position is given by $x(t)$ after time t seconds.

t	0	3	6	9
x(t)	0	2	1	5
v(t)	1.5	-1	3	4
a(t)	-2	4	2	4

a. Find displacement from 0 seconds to 9 seconds.

b. Find average velocity from 3 seconds to 9 seconds.

c. Find average acceleration from 0 seconds to 6 seconds.

4.3 RATES OF CHANGE IN APPLIED CONTEXT OTHER THAN MOTION

1. Given the area of a circle $A = \pi r^2$, whose radius is r,

    a. Find rate of change of the area with respect to the radius.

    b. How fast is the circle's area changing when $r = 5$ cm?

    c. Interpret the answer to b.

2. The cost of producing m cellphones is $c(m) = 300 + 100m - 0.2m^2$.

    a. Find the average cost of producing 100 cellphones and explain what the answer represents.

    b. Find instantaneous rate of change when 50 cellphones are produced. Explain what the answer represents.

4.4 INTRODUCTION TO RELATED RATES

1. A circle's radius and area are related by $A = \pi r^2$. If the circle is growing, write an equation that relates the rate of change of the area A with

respect to time t, to the rate of change of the radius r with respect to time t.

2. Mulch is being dropped from a dump truck at a rate of 20 cubic feet per minute, and it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 5 feet high? In the context of this problem, match each value to a description.

Value

a. Height and base of the cone.

b. Rate at which mulch is being dumped.

c. Rate at which the pile is growing when the height of the pile is 5 feet.

Description

- Quantities that vary depending on time
- Quantity that's not changing
- Quantity that exists at only a specific moment in time

3. A 10-foot ladder is leaning against a house when the base starts to slide away. When the base is 8 feet from the house, it's sliding away at 3 ft/sec. Fill in the blanks with "constant," "positive," or "negative."

$\frac{dx}{dt}$, the distance along the ground between the house and the base of the ladder, is ________________,

$\frac{dy}{dt}$, the distance along the wall between the top of the ladder and the ground, is ________________, and

$\frac{dz}{dt}$, the length of the ladder, is ________________.

4.5 SOLVING RELATED RATES PROBLEMS

1. A 10-foot ladder is leaning against a house when the base starts to slide away. When the base is 8 feet from the house, it's sliding away at 3 ft/sec. How fast is the ladder sliding down the wall at this moment?

2. A spherical balloon is inflated at a rate of 25 cm^3/sec. How fast is the balloon's radius increasing when the radius is 5 cm. The volume of a sphere is given by $V=\frac{4}{3}\pi r^3$.

3. Let $y=f(x)=4x^2-2x+1$. If $\frac{dx}{dt}=-3$ cm/sec, complete the table.

x	0	1	2
dy/dt			

4.6 APPROXIMATING VALUES OF A FUNCTION USING LOCAL LINEARITY AND LINEARIZATION

1. For $f(x) = x^3 - 4$, complete the table.

x	-2			
f(x)				23
f'(x)		0		
Equation of the tangent line	y+12=12(x+2)		y+3=3(x-1)	

2. Write an equation for the line tangent to $f(x) = x^{\frac{1}{3}}$ at $x = 8$, and use it to approximate $f(8.1)$.

4.7 USING L'HOSPITAL'S RULE FOR DETERMINING LIMITS OF INDETERMINATE FORMS

1. Circle the limits that give indeterminate forms.

$$\lim_{x\to 2}\frac{x-2}{x+2} \qquad \lim_{x\to 2}\frac{2-\sqrt{x^3-6}}{x-2} \qquad \lim_{x\to 2}\frac{x+2}{x-2} \qquad \lim_{x\to 0}\frac{e^x-\cos x}{x} \qquad \lim_{x\to\infty}\frac{e^x}{\ln x}$$

2. For continuous functions $f(x)$ and $g(x)$, with $\lim_{x\to 1} f(x) = 0$ and $\lim_{x\to 1} g(x) = 0$,

a. True or False? $\lim_{x\to 1}\frac{f(x)}{g(x)} = \frac{0}{0}$.

b. True or False? $\lim_{x\to1}\frac{f(x)}{g(x)}=\lim_{x\to1}\frac{g(x)f'(x)-f(x)g(x)}{[g(x)]^2}$

c. True or False? $\lim_{x\to1}\frac{f(x)}{g(x)}=\lim_{x\to1}\frac{f'(x)}{g'(x)}$

d. True or False? $\lim_{x\to2}\frac{f(x)}{g(x)}=\frac{f'(2)}{g'(2)}$

3. Let $\lim_{x\to0}\frac{\sin x}{x}=\lim_{x\to0}\frac{f(x)}{g(x)}$. Complete the table and fill in the statement.

	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
$\lim_{x\to0}$				

Because

$\lim_{x\to0}\sin x=$ ______ and $\lim_{x\to0}x=$ ______,

the given limit is indeterminate. Applying L'Hospital's Rule gives

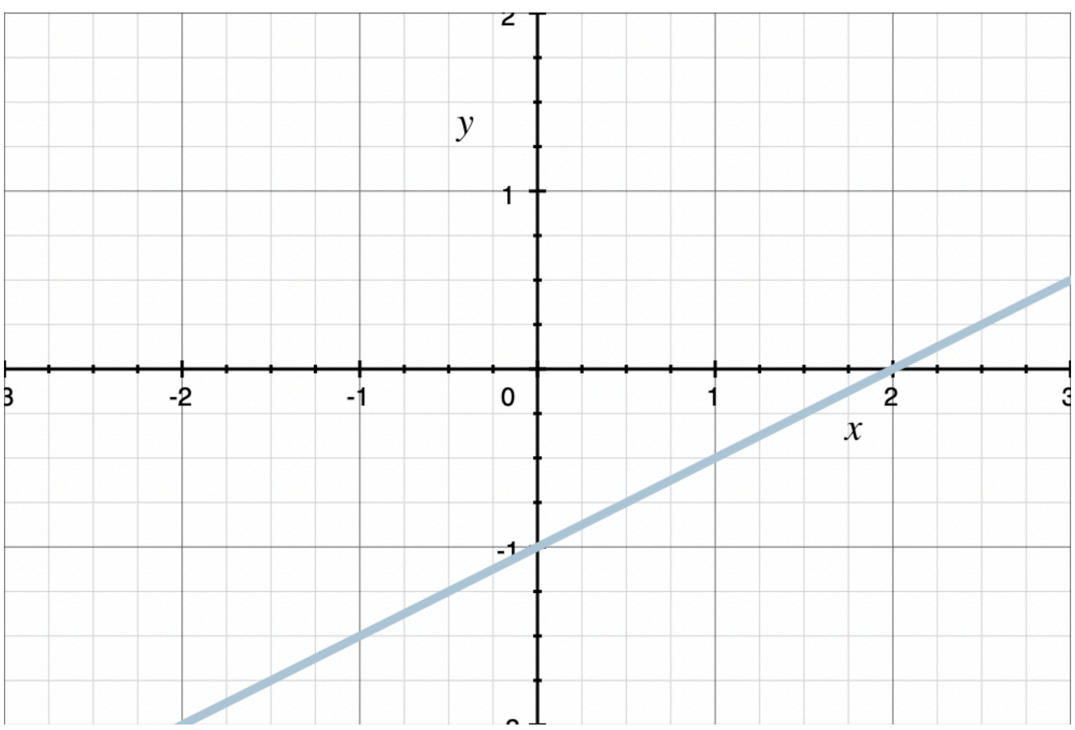
$$\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{f'(x)}{g'(x)}=\lim_{x\to 0}\frac{\quad}{\quad}=\_\_\_\_\_\_.$$

4. Because $\lim_{x\to 0}(e^x-\cos x)=$ ______ and $\lim_{x\to 0}x=$ ______, then L'Hospital's rule ______________ to $\lim_{x\to 0}\frac{e^x-\cos x}{x}$, and the value of the limit is

$$\lim_{x\to 0}\frac{e^x-\cos x}{x}=\lim_{x\to 0}\frac{\quad}{\quad}=\_\_\_\_\_\_.$$

5. Given the table of values for $f(x)$ and the graph of $g(x)$, for each limit, say whether or not L'Hospital's Rule applies, and find the limit.

x	0	2	4
f(x)	3	0	5
f'(x)	-2	4	0



a. $\lim_{x\to 2}\frac{f(x)}{g(x)}$

b. $\lim_{x\to 0}\frac{f(x)}{g(x)}$

6. Evaluate $\lim_{x\to 0}\frac{\sin^2 x}{x^2}$.