

8.1 FINDING THE AVERAGE VALUE OF A FUNCTION ON AN INTERVAL

1. Explain what $\frac{1}{b-a} \int_a^b f(x) dx$ calculates for the function f over the interval $[a, b]$.

2. Find the average value of $g(x) = x^2 + 4x - 1$ over the interval $[-1, 2]$.

3. The temperature (in degrees Fahrenheit) is modeled by

$$F(t) = 45 + 14 \sin\left(\frac{\pi t}{12}\right) \text{ where } t \text{ is the number of hours after } 8:00 \text{ a.m.}$$

Circle the integral below that would calculate the average temperature from 10:00 a.m. to 2:00 p.m.

$$\frac{1}{10-2} \int_2^{10} 45 + 14 \sin\left(\frac{\pi t}{12}\right) dt$$

$$\frac{1}{2-10} \int_{10}^2 45 + 14 \sin\left(\frac{\pi t}{12}\right) dt$$

$$\frac{1}{6-2} \int_6^2 45 + 14 \sin\left(\frac{\pi t}{12}\right) dt$$

$$\frac{1}{6-2} \int_2^6 45 + 14 \sin\left(\frac{\pi t}{12}\right) dt$$

4. If the average value of the function f over the interval $[a, b]$ is 3, find the value of $\int_a^b f(x) dx$.

8.2 CONNECTING POSITION, VELOCITY, AND ACCELERATION OF FUNCTIONS USING INTEGRALS

1. Let a position function be represented by $s(t)$, the velocity by $v(t)$, and the acceleration by $a(t)$ over an interval $[a, b]$. Match one integral to each of the following expressions.

$$\int_a^b a(t) dt \quad \frac{\int_a^b a(t) dt}{b-a} \quad \frac{1}{b-a} \int_a^b v(t) dt \quad s(a) + \int_a^b v(t) dt \quad \int_a^b v(t) dt$$

a. $s(b) - s(a) =$

b. $\frac{s(b) - s(a)}{b-a} =$

c. $v(b) - v(a) =$

d. $\frac{v(b) - v(a)}{b-a} =$

e. $s(b) =$

2. Circle the correct position function, $x(t)$, for a particle with velocity $v(t) = \sin t + 2$, if the position of the particle at $t = 0$ is 1.

a. $x(t) = \cos t + 2t + 2$

b. $x(t) = -\cos t + 2t$

c. $x(t) = -\cos t + 2t + 2$

d. $x(t) = -\cos t + 2$

8.3 USING ACCUMULATION FUNCTIONS AND DEFINITE INTEGRALS IN APPLIED CONTEXTS

1. Karen is hiking, at a rate given by the differentiable function $m(t)$, measured in feet per minute. Match each of the following integrals with an expression below.

$$\frac{1}{30} \int_0^{30} m'(t) dt \quad \int_0^{30} m'(t) dt \quad \int_0^{30} m(t) dt \quad \int_0^{30} |m(t)| dt \quad \frac{1}{30} \int_0^{30} m(t) dt$$

Karen's average rate of change over the first 30 minutes.

The change in Karen's pace over the first 30 minutes.

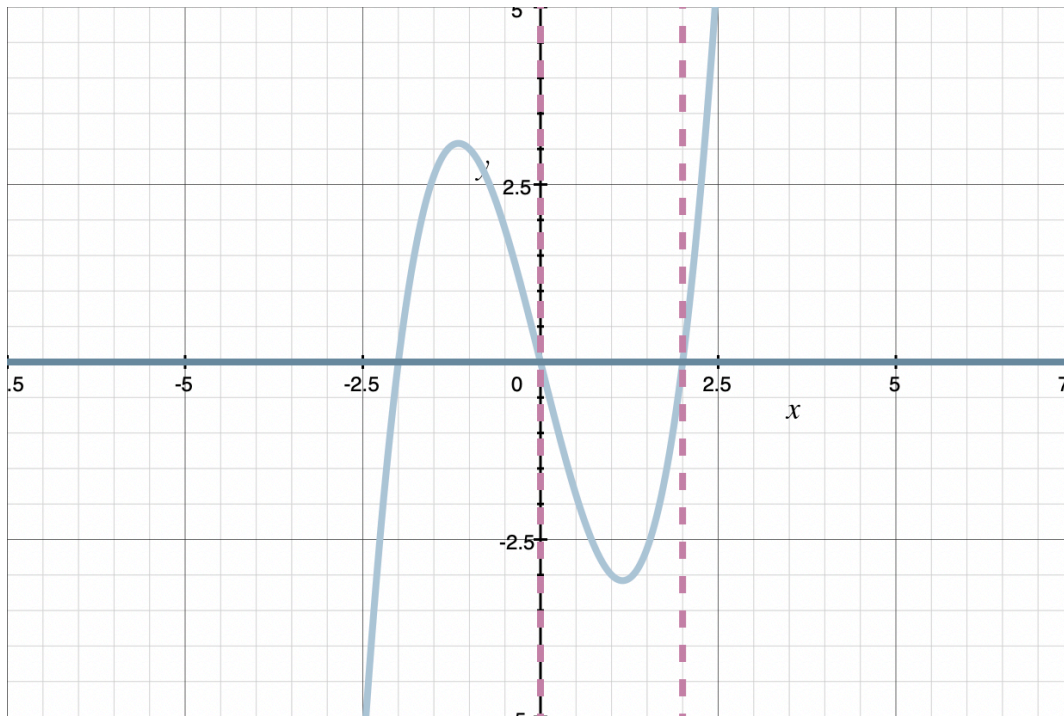
Karen's displacement over the first 30 minutes.

The average change in Karen's pace over the first 30 minutes.

The total distance Karen hikes over the first 30 minutes.

8.4 FINDING THE AREA BETWEEN CURVES EXPRESSED AS FUNCTIONS OF X

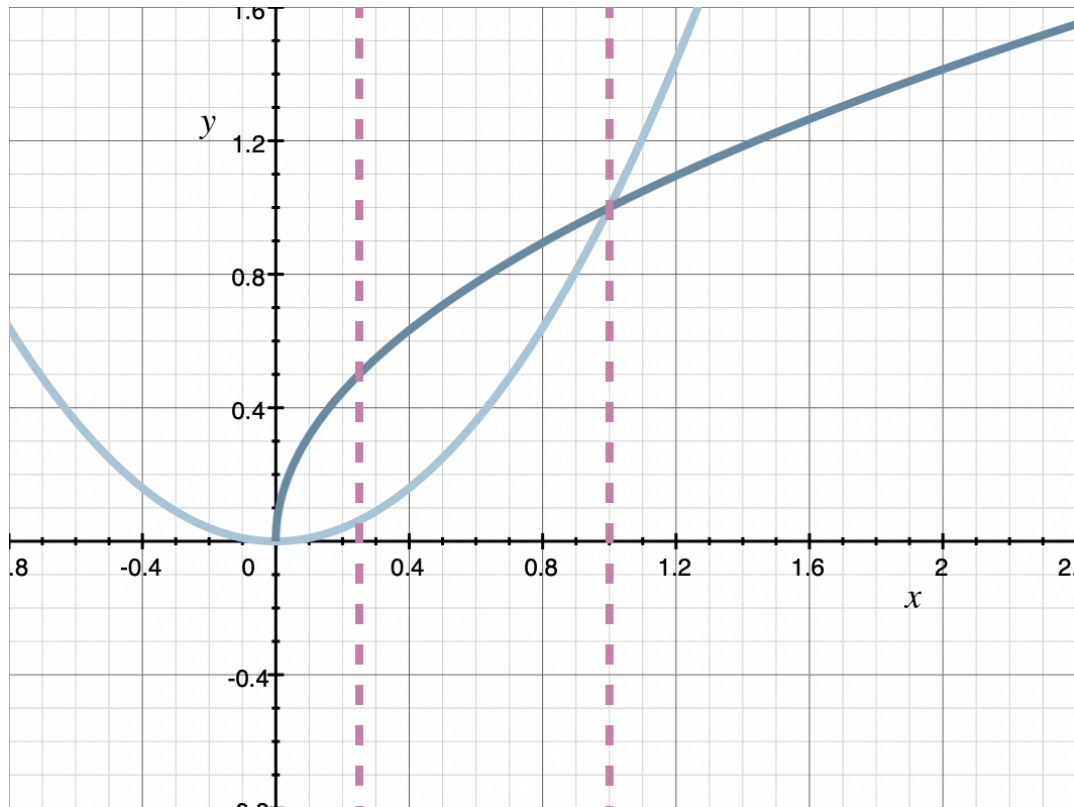
1. The graph below shows the region R bounded by $y = x^3 - 4x$, $x = 0$, $y = 0$, and $x = 2$. Set up and evaluate an integral for the area of R .



We can find the area of the region R by summing the lengths of _____ (horizontal, vertical) strips, from $x =$ _____ to $x =$ _____. The function that defines the top of the region is _____, and the function that defines the bottom is _____. To find the area, we need to calculate the integral $\int_0^2 [(\text{_____}) - (\text{_____})] dx$.

2. Draw a sketch of the region R bounded by the given curves and lines below. Set up and evaluate an integral to find the area of the region R .

$$y = x^2, y = \sqrt{x}, x = \frac{1}{4}, x = 1$$



We can find the area of the region by summing the lengths of _____ (horizontal, vertical) strips, from $x =$ _____ to $x =$ _____. The function that defines the top of the region is _____ and the function that defines the bottom is _____. To find the area, we need to calculate the integral

$$\int_{\frac{1}{4}}^1 [(\text{_____}) - (\text{_____})] dx.$$

3. Find a vertical line $x = k$ that divides the area enclosed by $y = x^2$, $y = 0$, and $x = 2$ into two equal parts.

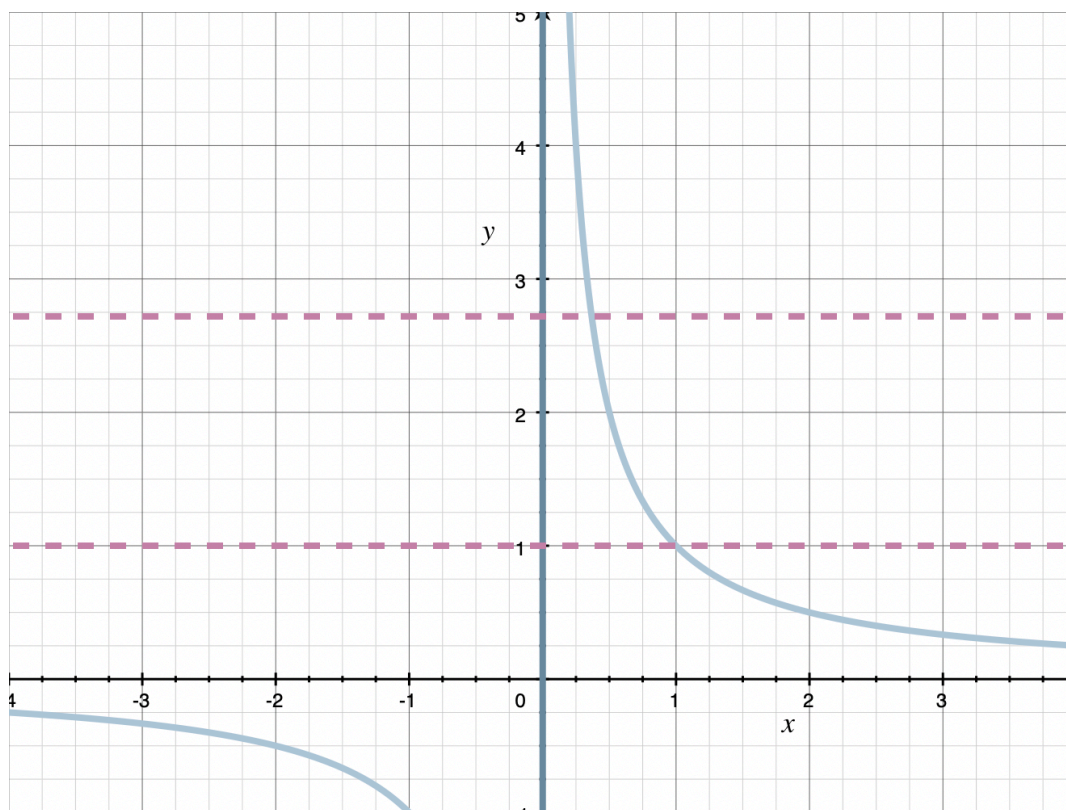
a. Sketch the region.

b. Set up an integral to find total area, then evaluate the integral.

- c. Find a line $y = k$ that divides the area of the region in half.

8.5 FINDING THE AREA BETWEEN CURVES EXPRESSED AS FUNCTIONS OF Y

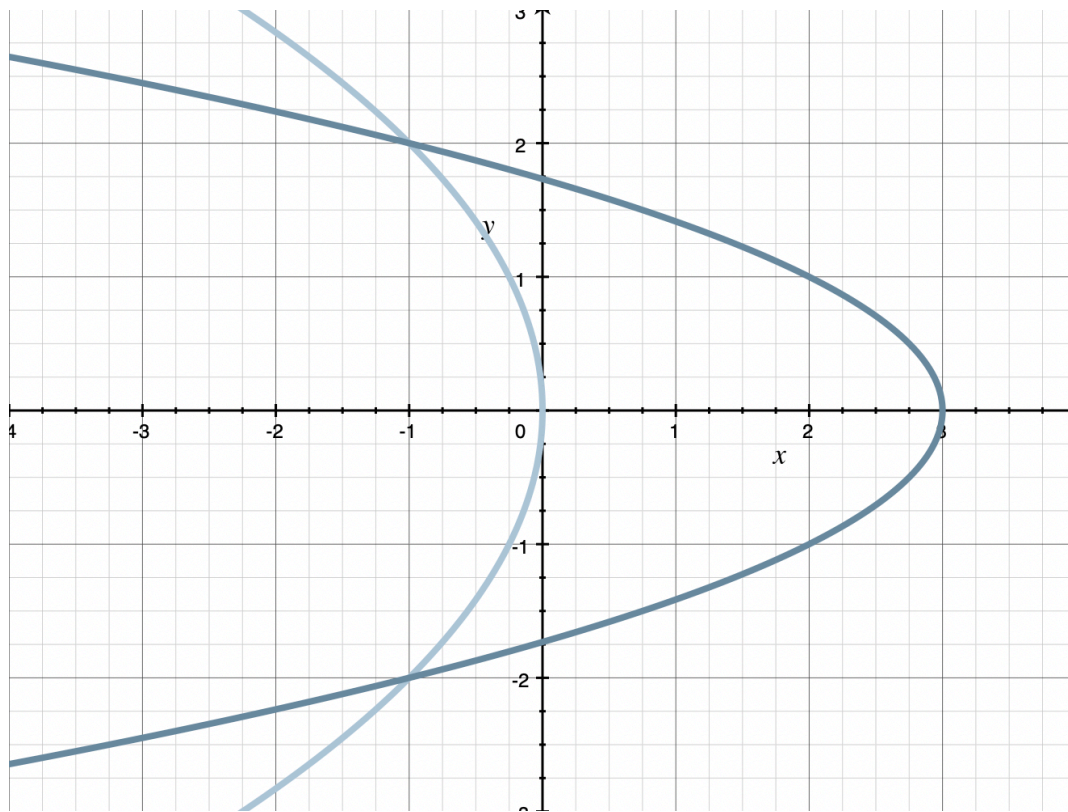
1. The graph below shows the region R bounded by $x = \frac{1}{y}$, $x = 0$, $y = 1$, and $y = e$. Set up and evaluate an integral that calculates the area of the region.



We can find the area of the region by summing the lengths of _____ (horizontal, vertical) strips, from $y = 1$ to $y = e$. The function that defines the right side of our region is _____, and the function that defines the left side is _____. To find the

area, we need to calculate the integral $\int_1^e [(\text{_____}) - (\text{_____})] dy$.

2. The graph below shows the region R bounded by $y^2 = -4x$ and $x = 3 - y^2$.



The points of intersection of the curves are _____ and _____. We can find the area of the region by summing the lengths of _____ (horizontal, vertical) strips, from _____ to _____. The function that defines the right side of the region is _____ and the function that defines the left side of the region is _____. To find the area, we need to calculate the integral $\int_{-2}^2 [(\text{_____}) - (\text{_____})] dy$.

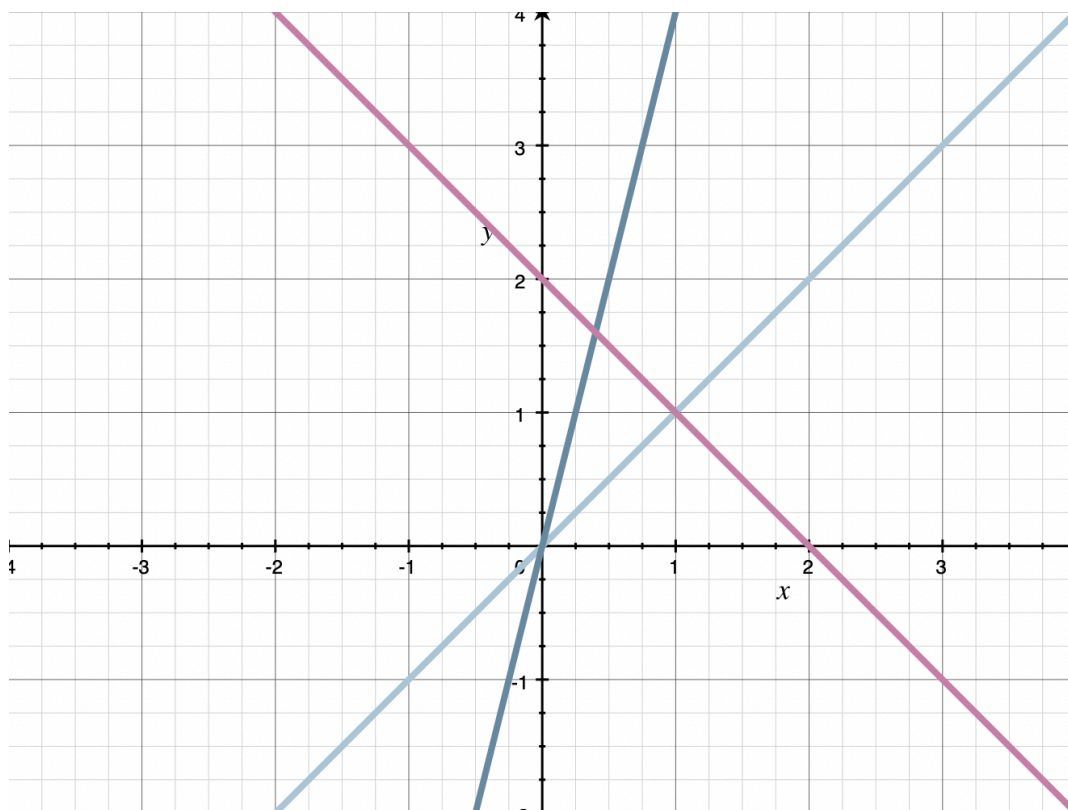
3. Sketch the graphs of $x^2 = y$, $y = 0$, and $x = 2$. The area of the region they enclose can be found using both functions of x and functions of y . Below

is the integral in terms of x . Write the integral in terms of y for the same region, then verify that it produces the same area.

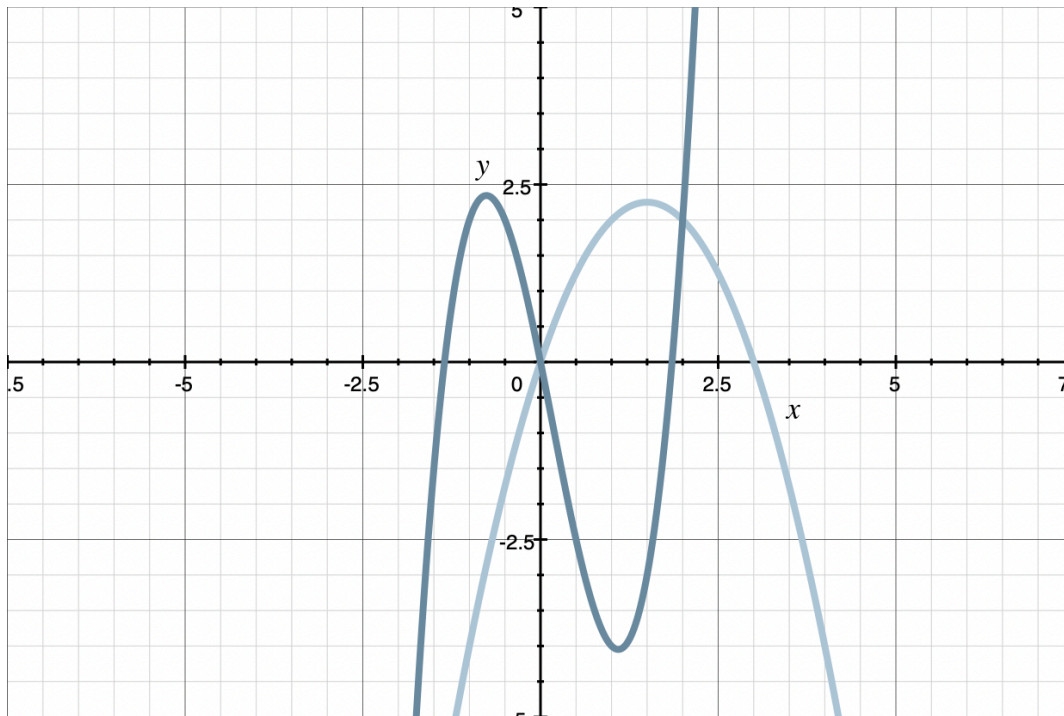
$$\int_0^2 x^2 dx = \frac{8}{3}$$

8.6 FINDING THE AREA BETWEEN CURVES THAT INTERSECT AT MORE THAN TWO POINTS

1. Set up and evaluate the integrals that will calculate the area for the region below bounded by $y = x$, $y = 4x$, and $y = -x + 2$.

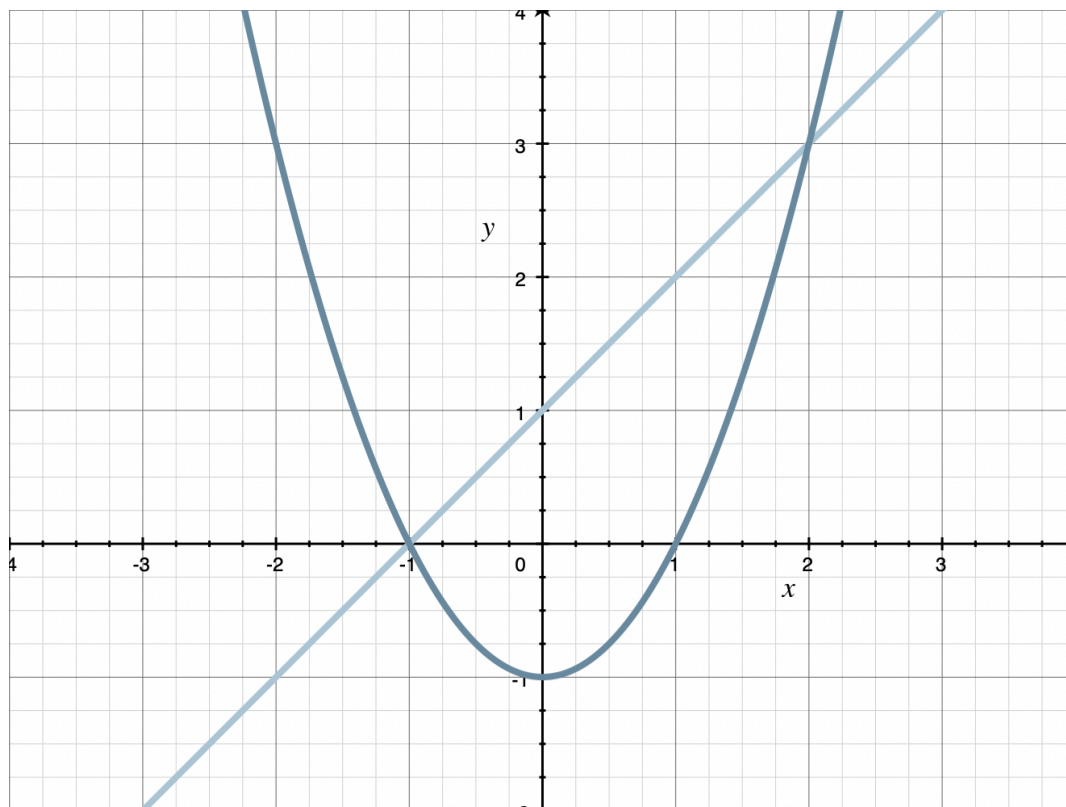


2. The graphs of $y = -x^2 + 3x$ and $y = 2x^3 - x^2 - 5x$ intersect at points $(-2, -10)$, $(0,0)$, and $(2,2)$. Set up the integrals that will find the area of both of the enclosed regions that these functions form.



8.7 VOLUMES WITH CROSS SECTIONS: SQUARES AND RECTANGLES

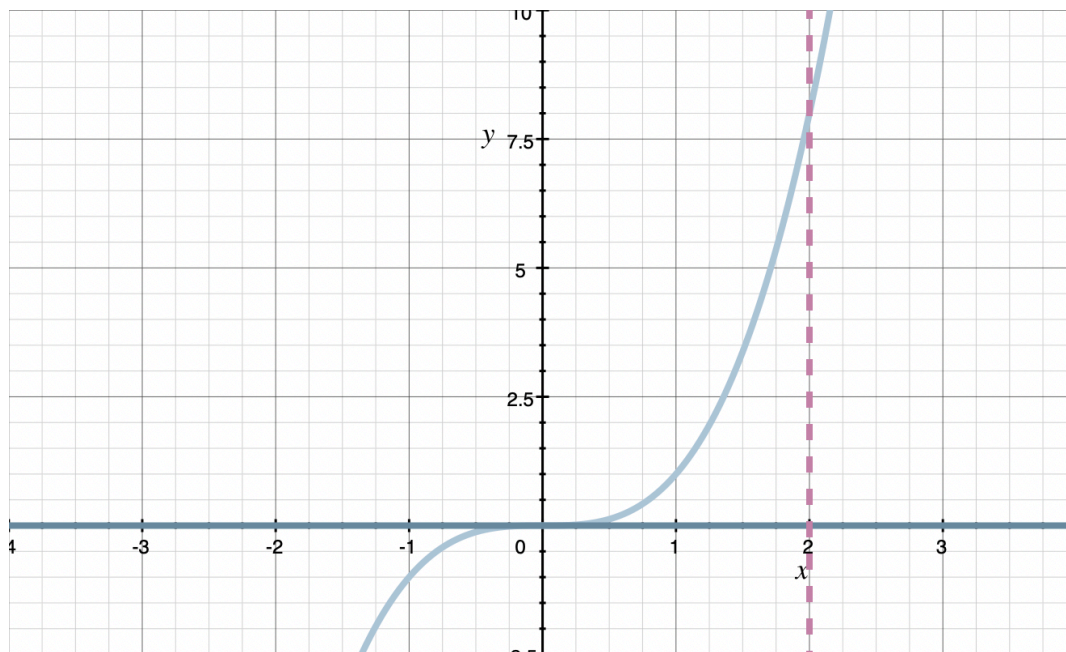
1. Set up the integral that would find the volume of the solid whose base is bounded by The base of a solid is bounded by $y = x + 1$ and $y = x^2 - 1$. Set up an integral to find the volume of the solid using cross sections perpendicular to the x -axis,



a. when the cross sections are squares.

b. when the cross sections are rectangles with height 2.

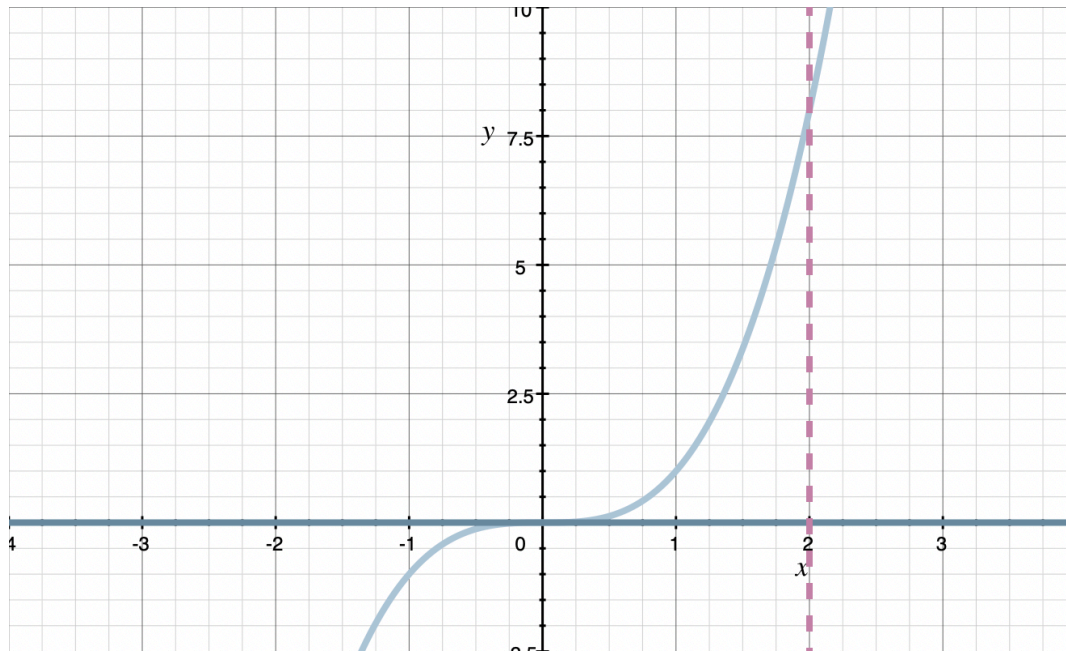
2. The graph below shows a region bounded by $y = x^3$, $x = 2$, and $y = 0$, which forms the base of some solid. Match each integral with the statement it represents.



- The volume given by square cross sections that are perpendicular to the x -axis:
- The volume given by square cross sections that are perpendicular to the y -axis:
- The volume given by rectangular cross sections that are perpendicular to the x -axis with height 1:
- The volume given by rectangular cross sections that are perpendicular to the y -axis with height 1:

8.8 VOLUMES WITH CROSS SECTIONS: TRIANGLES AND SEMICIRCLES

- The graph below shows a region bounded by $y = x^3$, $x = 2$, and $y = 0$, which forms the base of a solid. Match each integral with the statement it represents.

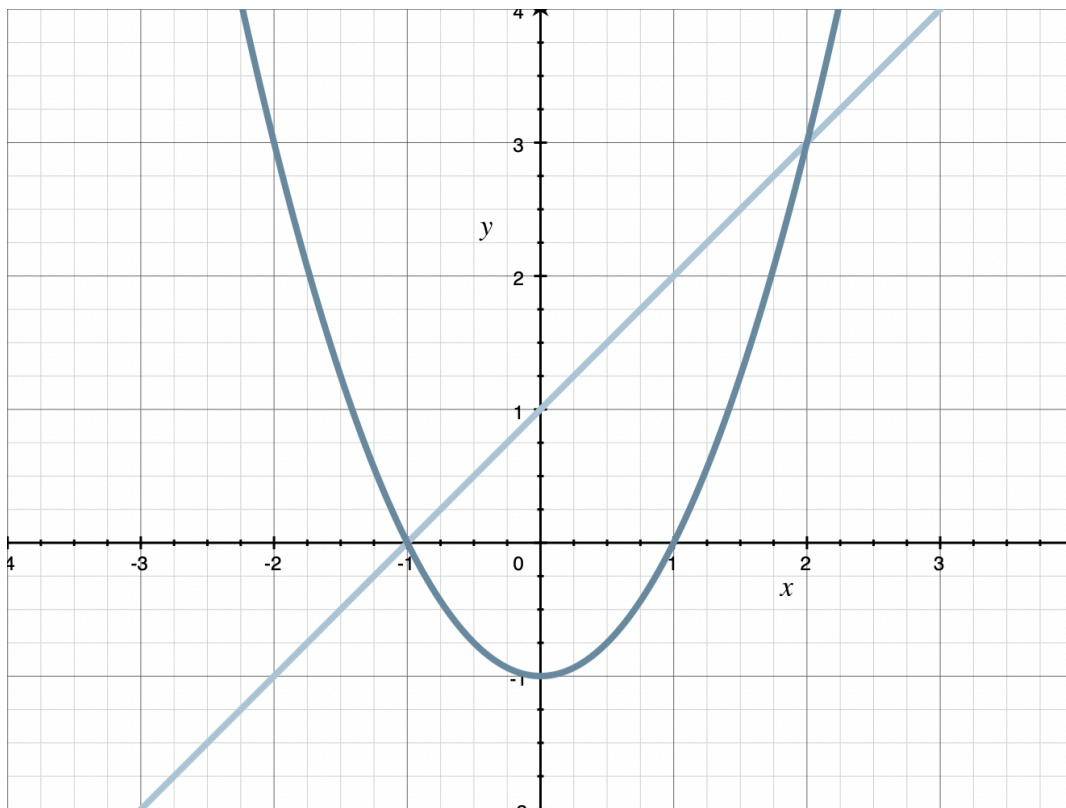


- a. Equilateral triangles with the base perpendicular to the x -axis:
- b. Equilateral triangles with the base perpendicular to the y -axis:
- c. Isosceles right triangles with the leg as the base of the solid perpendicular to the x -axis:
- d. Isosceles right triangles with the leg as the base of the solid perpendicular to the y -axis:
- e. Isosceles right triangles with the hypotenuse as the base of the solid perpendicular to the x -axis:
- f. Isosceles right triangles with the hypotenuse as the base of the solid perpendicular to the y -axis:

g. Semicircles with the the diameter as the base perpendicular to the x -axis:

h. Semicircles with the the diameter as the base perpendicular to the y -axis:

2. Set up the integral that calculates the volume of the solid whose base is bounded by $y = x + 1$ and $y = x^2 - 1$, using cross sections that are perpendicular to the x -axis that are:



a. Equilateral triangles:

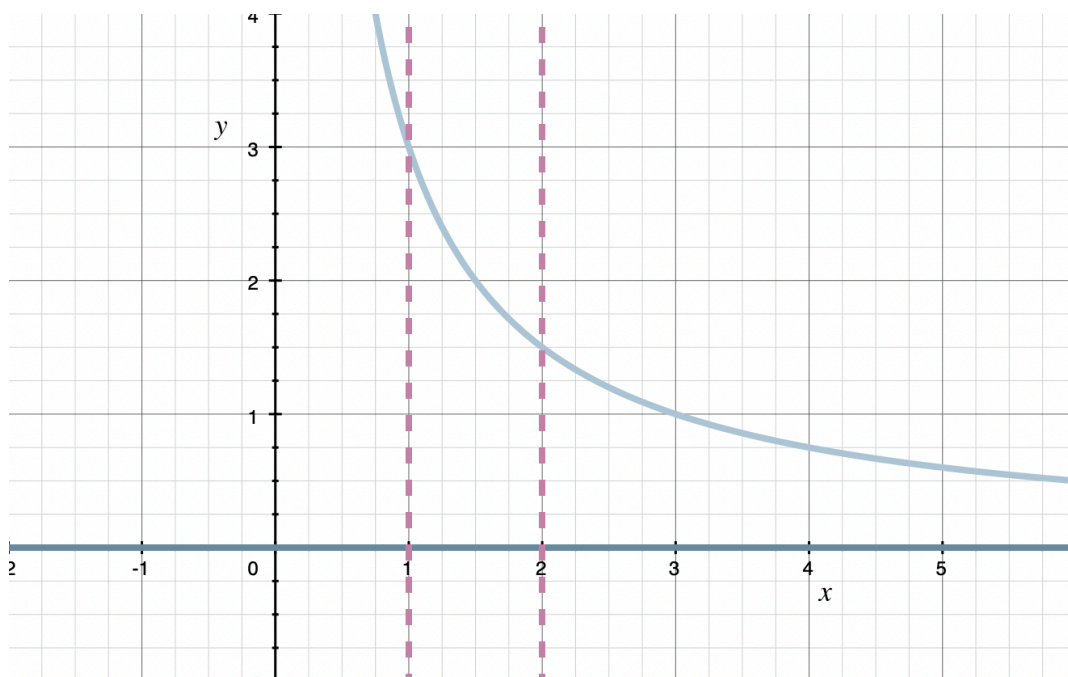
b. Isosceles right triangles with the leg as the base of the solid:

c. Isosceles right triangles with the hypotenuse as the base of the solid:

d. Semicircles with the diameter as the base:

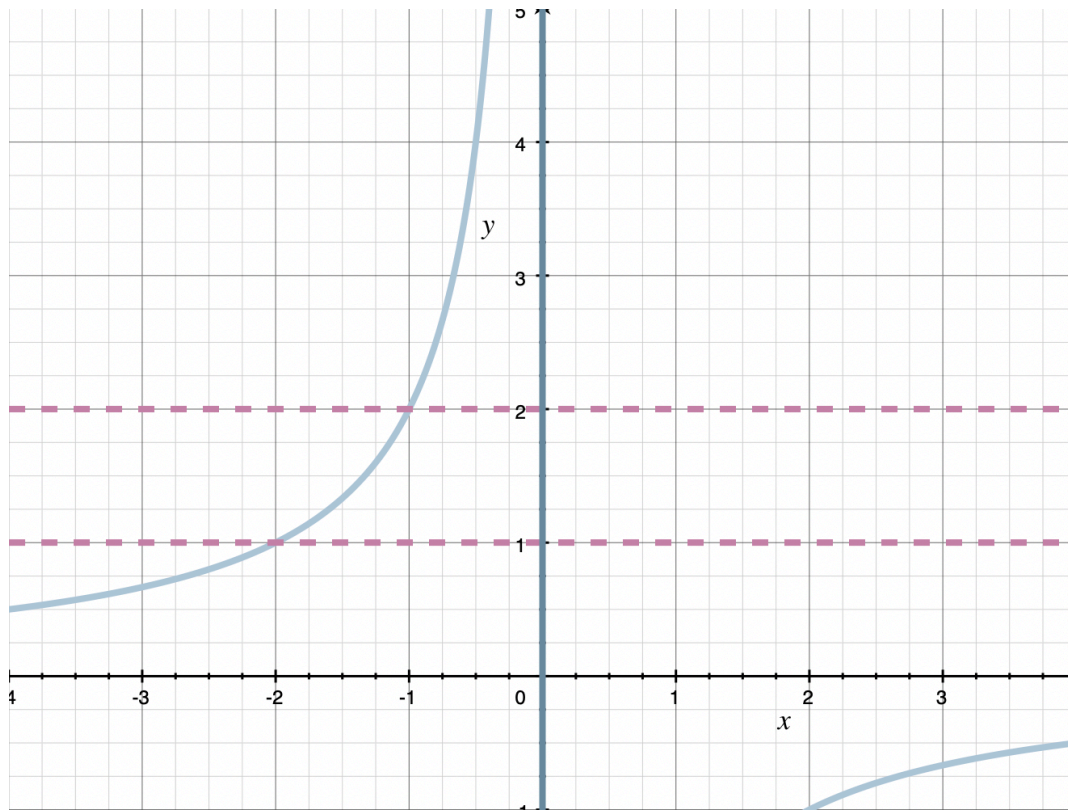
8.9 VOLUME WITH DISC METHOD: REVOLVING AROUND THE X- OR Y-AXIS

1. The graph shows the region enclosed by $g(x) = \frac{3}{x}$, $x = 1$, $x = 2$, and $y = 0$. Set up an integral that would find the volume of this region rotated around the x -axis.



2. The graph shows the region enclosed by $x = -\frac{2}{y}$, $y = 1$, $y = 2$, and $x = 0$.

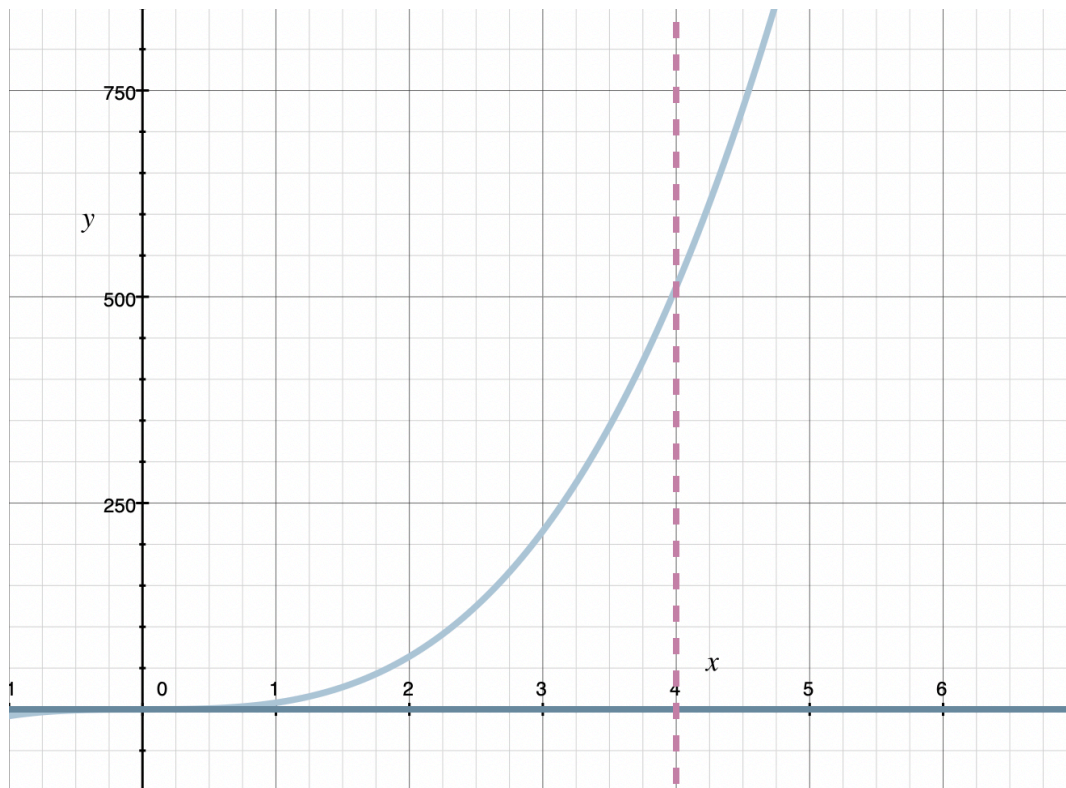
Set up an integral that would find the volume of this region rotated around the y -axis.



8.10 VOLUME WITH DISC METHOD: REVOLVING AROUND OTHER AXES

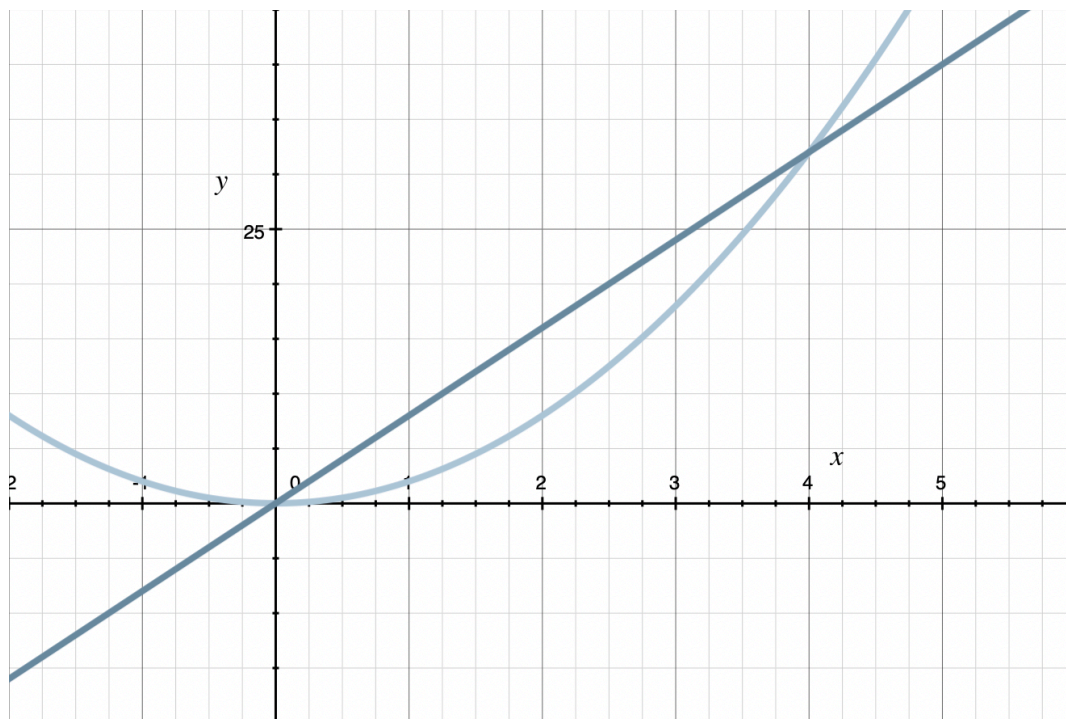
1. Sketch the graphs of $f(x) = 3 - x^2$ and $y = -1$, then set up an integral to find the volume when this region is rotated around $y = -1$.

2. The graph shows the shaded region enclosed by $\frac{1}{2}y^{\frac{1}{3}} = x$, $x = 4$, and $y = 0$. Set up an integral to find the volume when the region is rotated around $x = 4$.



8.11 VOLUME WITH WASHER METHOD, REVOLVING AROUND THE X- OR Y-AXIS

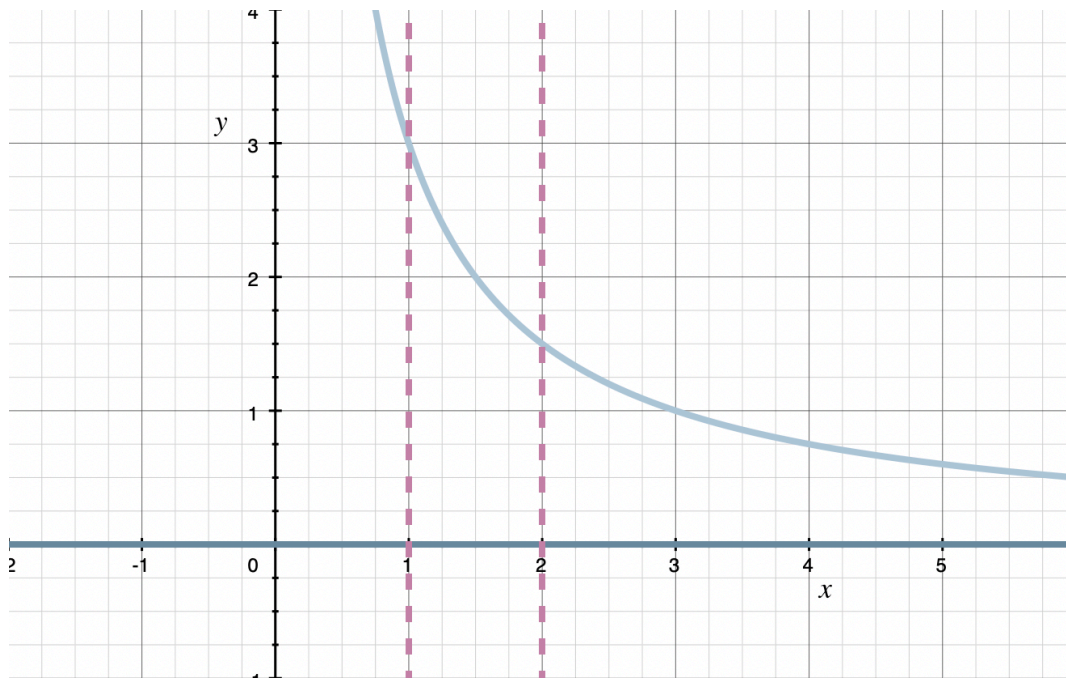
1. The graph below shows the region enclosed by $y = 2x^2$ and $y = 8x$.



- Set up the integral that would calculate the volume of the region when rotated around the x -axis.
- Set up the integral that would calculate the volume of the region when rotated around the y -axis.

8.12 VOLUME WITH WASHER METHOD: REVOLVING AROUND OTHER AXES

- The graph shows the region enclosed by $g(x) = \frac{3}{x}$, $x = 1$, $x = 2$, and $y = 0$.

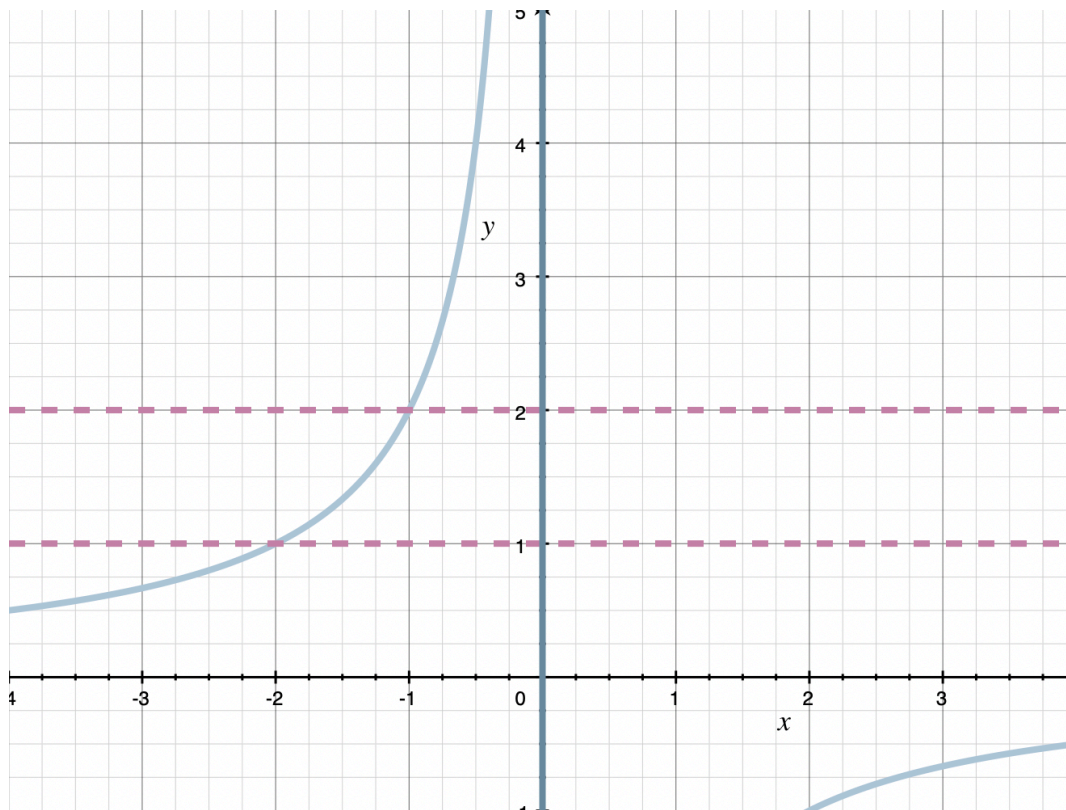


Set up an integral to calculate the volume when the region is rotated around,

a. the line $y = -2$:

b. the line $y = 6$:

2. The graph shows the region enclosed by $x = -\frac{2}{y}$, $y = 1$, $y = 2$, and $x = 0$.

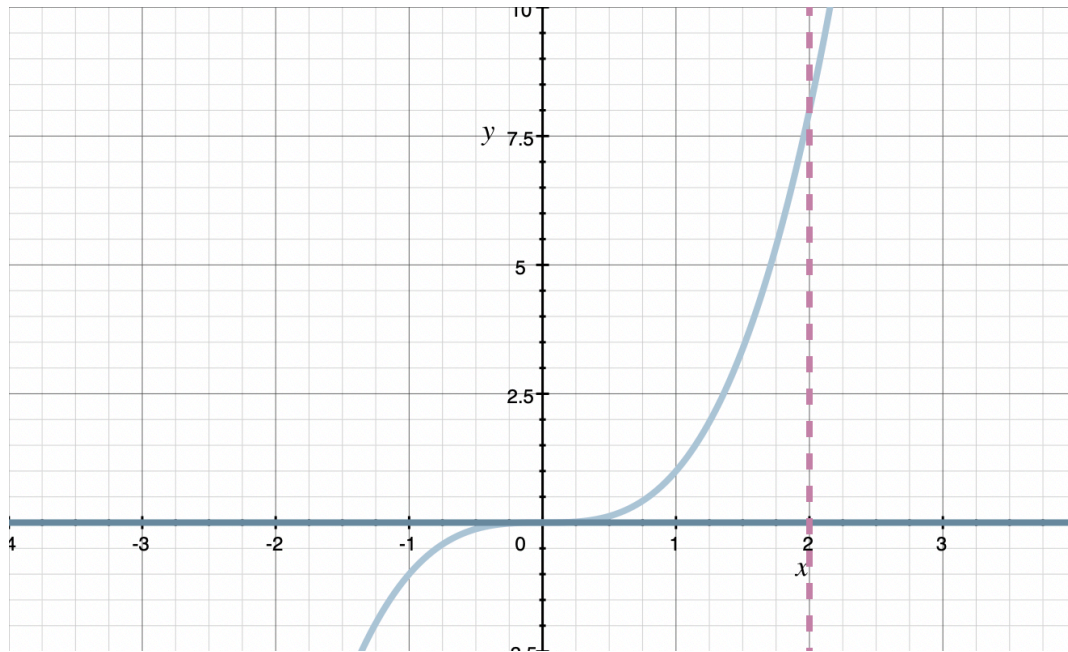


Set up an integral that would calculate the volume of the region rotated around,

a. the line $x = 2$:

b. the line $x = -4$:

3. The graph shows the base of a solid, which is bounded by $y = x^3$, $x = 2$, and $y = 0$. Explain what each integral represents.



a. $\pi \int_0^2 (x^3 - (-4))^2 - (0 - (-4))^2 dx$:

b. $\pi \int (5 - \sqrt[3]{y})^2 - (5 - 2)^2 dy$: