

## 1.1 INTRODUCING CALCULUS: CAN CHANGE OCCUR AT AN INSTANT?

1. If average rate of change is given by  $\Delta y/\Delta x$ , what is the only value of either  $\Delta y$  or  $\Delta x$  at which average rate of change is undefined?
2. Use the table to find average rate of change at  $x = 3$ .

x	0	2	4	6
f(x)	-7	6	4	-1

3. Change each expression for average rate of change to an expression for instantaneous rate of change.

$$\frac{f(x) - f(a)}{x - a}$$

$$\frac{f(x + h) - f(x)}{h}$$

## 1.2 DEFINING LIMITS AND USING LIMIT NOTATION

1. True or False? The limit exists at  $x = a$  and is a real number  $L$  if the value of the function  $f(x)$  gets closer and closer to  $L$  as we can move as close to  $a$  as we want, without arriving at exactly  $a$ .

## 1.3 ESTIMATING LIMIT VALUES FROM GRAPHS

1. The general limit exists at a point  $x = c$  if

a. \_\_\_\_\_

b. \_\_\_\_\_

c. \_\_\_\_\_

2. True or False? The general limit of  $f(x) = |x| + 3$  exists at  $x = -3$ . Why or why not?

3. Show that the limit  $\lim_{x \rightarrow 2} \frac{1}{x - 2}$  does not exist.

### 1.4 ESTIMATING LIMIT VALUES FROM TABLES

1. Use the values in the table to write an expression for the limit of  $f(x)$  at  $x = 5$ .

x	4.9998	4.9999	5	5.0001	5.0002
f(x)	5.9998	5.9999		6.0001	6.0002

2. Use the values in the table to write an expression for the limit of  $f(x)$  at  $x = -2$ .

x	-3	-2	-1	0	1
f(x)	10		8	7	6

## 1.5 DETERMINING LIMITS USING ALGEBRAIC PROPERTIES OF LIMITS

- The limit of a product is the \_\_\_\_\_ of the limits.
- The limit of a quotient is the \_\_\_\_\_ of the limits.
- Use properties of limits to evaluate each limit.

a.  $\lim_{x \rightarrow 4} \frac{1}{2}x^2 - \sqrt{x}$

b.  $\lim_{x \rightarrow 2} 3x^2 - 6x + 4$

c.  $\lim_{x \rightarrow \pi} \tan(3x)$

d.  $\lim_{x \rightarrow -1} (x^2 + 3x + 2)(x - 1)$

## 1.6 DETERMINING LIMITS USING ALGEBRAIC MANIPULATION

- Rewrite the function before evaluating the limit.

a.  $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x^2 - 9}$

b.  $\lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3}$

- Use algebraic manipulation to evaluate the trig limit.

a.  $\lim_{x \rightarrow 0} \frac{7}{x \csc x}$

b.  $\lim_{x \rightarrow 0} \frac{\cos x \sin x}{x}$

## 1.7 SELECTING PROCEDURES FOR DETERMINING LIMITS

1. Choose the correct procedure, then evaluate the limit.

a.  $\lim_{x \rightarrow -2} x^2 + 2x + 6$

b.  $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$

c.  $\lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h}$

## 1.8 DETERMINING LIMITS USING THE SQUEEZE THEOREM

1. The sine function oscillates back and forth on the interval \_\_\_\_\_.
2. The cosine function oscillates back and forth on the interval \_\_\_\_\_.
3. Which limits would we use to squeeze the limit?

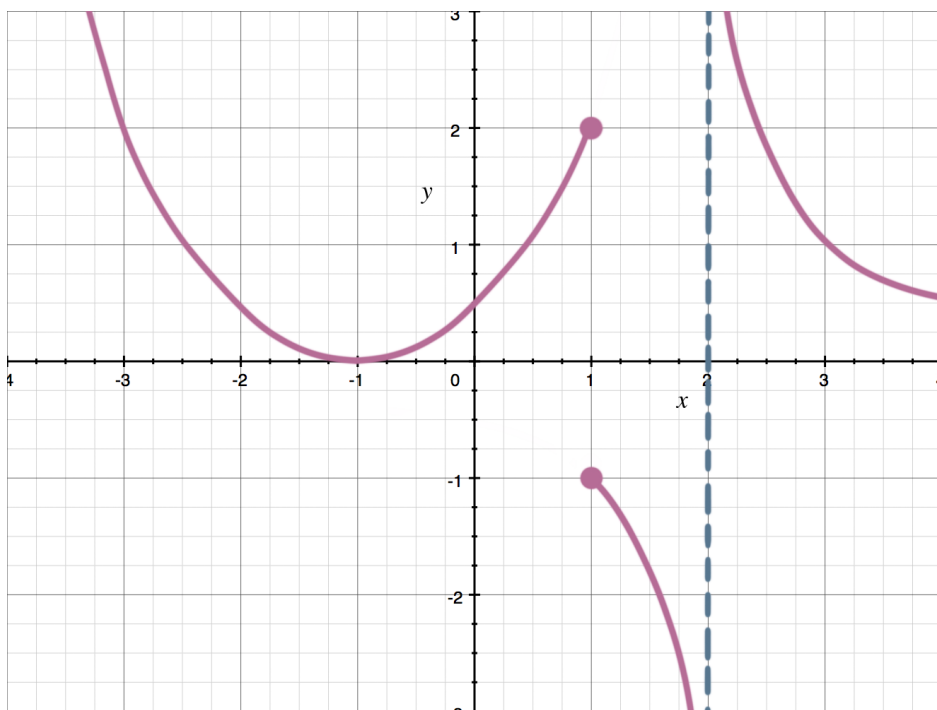
$$\leq \lim_{x \rightarrow 0} \frac{\sqrt{x} \cos x}{x} \leq$$

## 1.9 CONNECTING MULTIPLE REPRESENTATIONS OF LIMITS

- Show the following limit statement as an equation, a table, and a graph:  
“The limit as  $x$  approaches 4 of  $x^2$  is 16.”

## 1.10 EXPLORING TYPES OF DISCONTINUITIES

- Find and classify each discontinuity in the graph.



- Infinite discontinuities are also called \_\_\_\_\_ discontinuities.
- Point discontinuities are also called \_\_\_\_\_ discontinuities.
- True or False? A discontinuity always exists at a cusp in the graph.

## 1.11 DEFINING CONTINUITY AT A POINT

1. A function  $f$  is continuous at  $x = c$  if the following three conditions are met:

1. \_\_\_\_\_
2. \_\_\_\_\_
3. \_\_\_\_\_

2. What is the implication of the second condition?

## 1.12 CONFIRMING CONTINUITY OVER AN INTERVAL

1. List six types of functions that are continuous everywhere in their domains.
2. Which of the trig functions have their domains interrupted by horizontal asymptotes?
3. The domain of  $\ln x$  is \_\_\_\_\_.

## 1.13 REMOVING DISCONTINUITIES

1. Redefine the function as a continuous piecewise function.

$$f(x) = \frac{x^2 + 11x + 28}{x + 4}$$

2. Find the value of  $k$  that makes the function continuous.

$$f(x) = \begin{cases} k\sqrt{x+1} & 0 \leq x \leq 3 \\ 5-x & 3 < x \leq 5 \end{cases}$$

### 1.14 CONNECTING INFINITE LIMITS AND VERTICAL ASYMPTOTES

1. Where does the function have a vertical asymptote?

$$f(x) = \frac{1}{x-3}$$

2. If  $\lim_{x \rightarrow c^-} = \infty$  and  $\lim_{x \rightarrow c^+} = -\infty$ , does the general limit exist?

### 1.15 CONNECTING LIMITS AT INFINITY AND HORIZONTAL ASYMPTOTES

1. Does the function have a horizontal asymptote? If so, where?

$$f(x) = \frac{1}{x-3}$$

2. Describe the horizontal asymptote in each of the following cases.

N < D: \_\_\_\_\_

\_\_\_\_\_

N > D: \_\_\_\_\_

\_\_\_\_\_

N = D: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

3. Find any horizontal asymptote that exists for each of the following functions.

a.  $f(x) = \frac{x^3 - 2x^2}{x + 3}$

b.  $g(x) = \frac{x + 3}{x^3 - 2x^2}$

c.  $h(x) = \frac{x^3 + 3}{x^3 - 2x^2}$

### 1.16 WORKING WITH THE INTERMEDIATE VALUE THEOREM (IVT)

1. True or False? The IVT states that, assuming  $f(x)$  is continuous on  $[a, b]$ , if we can show that the function's value is negative at one side of the interval and positive at the other side, then that fact alone proves that the graph of the function must cross the  $x$ -axis at some point inside the interval.

2. Show that the function has at least one root in the interval  $[-2, 5]$ .

$$f(x) = x^3 - 4x^2 + 7x + 1$$