10.1 DEFINING CONVERGENT AND DIVERGENT INFINITE SERIES

1. Match the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ on the left with the corresponding solution on the right.

$S_n = 3 - \frac{5}{n^2}$	$a_3 = \frac{1}{8}$
$S_n = \frac{n-2}{n+5}$	$\sum_{n=1}^{3} a_n = \frac{7}{2}$
$S_n = 3 - n \cdot 2^{2-n}$	$a_1 = \frac{29}{8}$
$S_n = 3 + \frac{5n^2}{n^2+7}$	$\sum_{n=1}^{5} a_n = \frac{5,531}{720}$

2. An infinite series of numbers ________________ to a real number S if and only if the ________________ of its sequence of partial sums ________________ and is equal to ________________.

3. Say whether the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ converges or diverges. If it converges, calculate the value of S.

$$S_n = \frac{5}{2} + \frac{3n^2}{2n^2 - 1}$$

$$S_n = \frac{2}{n} - 5n^2$$

$$S_n = \frac{n^2 + 1}{(n-3)(2n+1)}$$

$$S_n = \frac{e^{2n}}{3 + e^{2n}}$$

$$S_n = \frac{e^n}{2n}$$

10.2 WORKING WITH GEOMETRIC SERIES

1. Circle any series below that are not geometric.

$$8 + 2 + \frac{2}{4} + \frac{2}{16} + \dots$$

$$\sum_{n=0}^{\infty} \frac{1}{n^4}$$

$$\sum_{n=0}^{\infty} \frac{3^n}{8^n}$$

$$\sum_{n=1}^{\infty} \frac{e^n}{2^{n-1}}$$

$$\sum_{n=0}^{\infty} \frac{n^4}{4^n}$$

$$8 - 12 + 36 - 48 + \dots$$

2. The geometric series $\sum_{n=0}^{\infty} ar^n$ ________________ to ________________ if ________________, and ________________ if ________________.

3. $\sum_{n=0}^{\infty} \frac{3^n}{2^{2n+3}}$

a. True or False? The series given is geometric.

b. True or False? The given series diverges.

c. If the series converges, find the sum.

4. For each series, find the constant ratio r, determine whether the series converges or diverges, and find the sum.

Series	r	Converges or diverges	Sum
$\sum_{n=0}^{\infty} \frac{5}{10^{n+1}}$			
$\sum_{n=1}^{\infty} 8^n$			
$\sum_{n=1}^{\infty} \frac{(-1)^n}{6^n}$			
$\sum_{n=1}^{\infty} \frac{e^n}{5^{n-1}}$			
$\sum_{n=0}^{\infty} 3(0.75)^n$			
$\sum_{n=0}^{\infty} (\ln 2)^{-n}$			

10.3 THE NTH TERM TEST FOR DIVERGENCE

1. If ________________, then $\sum_{n=1}^{\infty} a_n$ will ________________.

2. Evaluate the limit of each series and circle the ones that diverge.

$$\sum_{n=1}^{\infty} \frac{2n!}{3n!+1}$$

$$\sum_{n=1}^{\infty} \frac{(4n+5)(n+7)}{(n+1)^2(n-5)}$$

$$\sum_{n=1}^{\infty} \frac{3n^3-12n^2+5n-4}{2n^2-4n+7}$$

$$\sum_{n=1}^{\infty} \frac{2^n-5}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{\sin n}{n}$$

$$\sum_{n=1}^{\infty} 1-\frac{2}{\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \cos\left(\frac{2}{n}\right)$$

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n+7}$$

10.4 INTEGRAL TEST FOR CONVERGENCE

1. Let f be a ________________, ________________, ________________ function on $[1,\infty)$ and $a_n = f(n)$.

   If ________________ is convergent, then $\sum_{n=1}^{\infty} a_n$ is ________________. If ________________ is divergent, then $\sum_{n=1}^{\infty} a_n$ is ________________.

2. Circle the series below to which the integral test can be applied.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \qquad \sum_{n=1}^{\infty} \frac{1}{3^n}$$

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \qquad \sum_{n=0}^{\infty} \frac{5}{n^2+9}$$

$$\sum_{n=1}^{\infty} n^2 \qquad \sum_{n=1}^{\infty} \frac{1}{n}$$

3. True or false? Explain your answer. Determine whether the series converges or diverges.

   a. If $\sum_{n=1}^{\infty} 2^{-n}$, then $\lim_{a\to\infty} \frac{1}{2\ln 2} - \frac{1}{2^a \ln 2}$.

   b. If $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$, then $\lim_{a\to\infty} \frac{1}{2} \ln a$.

c. If $\sum_{n=1}^{\infty} \frac{4}{(5n-1)^2}$, then $\lim_{a\to\infty} \frac{1}{5} - \frac{4}{5(5a-1)}$.

10.5 HARMONIC SERIES AND P-SERIES

1. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, where p is any ________________ real number, is called a ________________. The ________________ $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if ________________ and divergent if ________________. If ________________, we call the resulting series the ________________ series, $\sum_{n=1}^{\infty} \frac{1}{n}$, which ________________.

2. Circle any series that converge by the p-series test.

$\sum_{n=1}^{\infty} \frac{1}{n}$	$\sum_{n=1}^{\infty} n^{\frac{4}{3}}$	$\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$	$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$
$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$	$\sum_{n=1}^{\infty} \frac{1}{n^{-3}}$	$\sum_{n=1}^{\infty} n^{-4}$	$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$

3. For each series, identify the value of p and use the p-series test to determine the convergence or divergence of each series.

Series	p	**Converges or diverges**
$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$		

$$\sum_{n=1}^{\infty} n^{-\frac{\pi}{2}}$$

$$\sum_{n=4}^{\infty} \frac{1}{n^7}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{0.8}}$$

$$\sum_{n=1}^{\infty} \frac{5}{n}$$

$$1 + \frac{1}{2\sqrt[3]{2}} + \frac{1}{3\sqrt[3]{3}} + \dots$$

4. True or False? Explain your answers.

a. The series $\sum_{n=1}^{\infty} \frac{n}{n^{3p-2}}$ converges for $p > \frac{4}{3}$.

b. The series $\sum_{n=1}^{\infty} \frac{1-\sqrt[3]{n}}{n^2}$ converges with $p = 2$.

c. The series $\sum_{n=1}^{\infty} \frac{n^2}{n^{2p+4}}$ is a harmonic series for $p = 1$.

10.6 COMPARISON TESTS FOR CONVERGENCE

1. Suppose a_n______, b_n______, and ____________ for all n. Then if ____________ converges, then ____________ converges, and if ____________ diverges, then ____________ diverges.

2. True or false? Explain your answer. Use the comparison test.

   a. Since $0 < \frac{1}{n^5+2} < \frac{1}{n^5}$ for any $n \geq 1$, then the series $\sum_{n=1}^{\infty} \frac{1}{n^5+2}$ converges.

   b. Since $0 < \frac{5}{\sqrt{26n}} < \frac{5}{\sqrt{25n}} = \frac{1}{\sqrt{n}}$ for any $n \geq 1$, then the series $\sum_{n=1}^{\infty} \frac{5}{\sqrt{26n}}$ diverges.

   c. Since $0 < \frac{1+\sin n}{10^n} < \frac{2}{10^n}$ for any $n \geq 1$, then $\sum_{n=0}^{\infty} \frac{1+\sin n}{10^n}$ diverges.

3. Match each of the following comparison series to the correct series in the table below. Then determine whether each series from the second column converges or diverges and the reason for it. Finally, determine convergence or divergence of the original series.

$$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n \quad \sum_{n=1}^{\infty} e^{-n} \quad \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}} \quad \sum_{n=1}^{\infty} \frac{1}{n^4} \quad \sum_{n=1}^{\infty} \left(\frac{5}{8}\right)^n \quad \sum_{n=1}^{\infty} \frac{1}{n} \quad \sum_{n=1}^{\infty} \frac{1}{n}$$

Series	Comparison	Con/Diverges	Con/Diverges
$\sum_{n=1}^{\infty} \frac{e^{-n}}{n+\cos^2 n}$			
$\sum_{n=1}^{\infty} \frac{5^n}{8^n+1}$			
$\sum_{n=1}^{\infty} \frac{\sqrt{n^6-1}}{n^7+7}$			
$\sum_{n=1}^{\infty} \frac{2^{\frac{1}{n}}}{n}$			
$\sum_{n=1}^{\infty} \frac{n^2-8n}{n^3+2n+1}$			
$\sum_{n=1}^{\infty} \frac{n^2-1}{n^2\sqrt{n}}$			
$\sum_{n=1}^{\infty} \frac{n+2^n}{n+6^n}$			

4. Suppose we have two series $\sum a_n$ and $\sum b_n$ with a_n______ and b_n______ for all n. $L = \lim_{n\to\infty}$ ______. If L is ____________ and ____________, then either ____________ ____________ converge or ____________ ____________ diverge.

5. What steps do we take to apply limit comparison test?

1. ____

2. ____

3. ____

4. ____

6. For each series, find a comparison series whose convergence can be easily determined. Then find the limit and using the limit comparison test, determine whether each series converges or diverges.

Series	Second series	$\lim_{n\to\infty} \frac{a_n}{b_n}$	Con/Diverges
$\sum_{n=2}^{\infty} \frac{5n^3+n}{2\sqrt{n^9+n^2}}$			
$\sum_{n=1}^{\infty} \frac{1}{n^2-8}$			
$\sum_{n=1}^{\infty} \frac{5+3^n}{2^n}$			
$\sum_{n=1}^{\infty} \frac{n+5}{(n+1)^3}$			
$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n-1}$			
$\sum_{n=1}^{\infty} \frac{1}{3n+5}$			

10.7 ALTERNATING SERIES TEST FOR CONVERGENCE

1. An alternating series $\sum (-1)^n a_n$ or $\sum (-1)^{n+1} a_n$ converges if __________ and if a_n is ______________ and ______________.

2. For each series, determine whether or not $a_n > 0$, and whether or not a_n is increasing or decreasing. Then find the limit of a_n and determine whether the series converges or diverges.

Series	a_n	In/Decreasing	$\lim_{n\to\infty} a_n$	Con/Diverges
$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n+1}$				
$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$				
$\sum_{n=1}^{\infty} \frac{(-1)^n\ n^2}{2n^2+5}$				
$\sum_{n=0}^{\infty} \frac{1}{(-1)^n(3^n+4^n)}$				
$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}\ e^n}{n^2}$				

3. Match the nth partial sum of the series on the left to the values of p on the right that cause the series to converge.

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{p+7}}$ $\qquad\qquad$ $p > -6$

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{5}{p}\right)^n \qquad p < 0$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} (3n)^p \qquad p > 5$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} (5p)^n \qquad p < \frac{1}{5}$$

10.8 RATIO TEST FOR CONVERGENCE

1. Let $\sum a_n$ be a ________________ series and $\lim_{n\to\infty}$ ___________ $= L$.

   - if ___________, then $\sum a_n$ converges.
   - if ___________ or if L is ___________, then $\sum a_n$ diverges.
   - if ___________, then there is no conclusion.

2. Complete the table.

Series	a_{n+1}	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert$	**Con/Diverges/Inconclusive**
$\sum_{n=1}^{\infty} \frac{n^3}{(n+1)!}$			
$\sum_{n=1}^{\infty} \frac{n^{12}}{5^n}$			

$$\sum_{n=1}^{\infty} \frac{n(n+2)}{(n+4)(n+5)}$$

$$\sum_{n=1}^{\infty} \frac{(5-n)\ 7^n}{3 \cdot 5^n}$$

$$\sum_{n=1}^{\infty} \frac{n^5}{2^{n^2}}$$

$$\sum_{n=1}^{\infty} \frac{n!}{7^n}$$

10.9 DETERMINING ABSOLUTE OR CONDITIONAL CONVERGENCE

1. A series $\sum a_n$ is called absolutely convergent if $\sum |a_n|$ is ____________. If $\sum a_n$ is ____________ and $\sum |a_n|$ is ____________, then the series is conditionally convergent.

2. True or False? Explain your answer.

   a. $\sum_{n=1}^{\infty} (-1)^{n+1} n^{-\frac{1}{5}}$ is absolutely convergent.

   b. $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ is conditionally convergent.

c. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+7}$ diverges.

10.10 ALTERNATING SERIES ERROR BOUND

1. Complete the sentence about the error of the approximation.

a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3-3}$ is approximated using the partial sum $\sum_{n=1}^{25} \frac{(-1)^{n+1}}{2n^3-3}$. The error bound is ________________ and so the approximation is an ________________.

b. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n3^n}$ is approximated using the partial sum $\sum_{n=1}^{16} \frac{(-1)^{n+1}}{n3^n}$. The error bound is ________________ and so the approximation is an ________________.

2. Find S_n as the approximation to the sum of the infinite series using the given number of terms. Set up an inequality to determine the maximum error for the approximation and find the maximum error. Then find an interval where the sum converges.

Series	S_n	**Max error inequality**	**Closed interval**
$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ n}{3^n}$ (4)			
$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n!}$ (6)			

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\,5}{\sqrt{n}}$ (3)

$\sum_{n=1}^{\infty} \frac{(-1)^{n}\sqrt{2n+5}}{\sqrt{n^2+1}}$ (10)

$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\,2}{3n^2}$ (5)

3. Set up an inequality to determine the minimum number of terms needed to find the sum of the infinite series with an error less than 0.01. Solve each inequality to find the minimum number of terms needed.

a. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$, then $|R_n| \leq \frac{1}{2n} \leq 0.01$.

b. $\sum_{n=0}^{\infty} \frac{5(-1)^n}{3n+1}$, then $|R_n| \leq \frac{5}{3n+4} \leq 0.01$.

c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$, then $|R_n| \leq \frac{1}{n^4+1} \leq 0.01$.

10.11 FINDING TAYLOR POLYNOMIAL APPROXIMATIONS OF FUNCTIONS

1. A power series about $x = a$ is a ___________ of ___________ multiplied by powers of ___________.

2. Match the following four expressions to its corresponding statement below.

$$\frac{f^n(a)}{n!} \qquad \frac{f^n(0)}{n!} \qquad x = 0$$

$$P_n(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \dots$$

a. Taylor polynomials have the following form.

b. The coefficient of the nth degree term in a Taylor polynomial for a function f centered at $x = a$.

c. Maclaurin polynomials are Taylor series that are used to approximate function values about.

d. The coefficient of the nth degree term in a Taylor polynomial for a function f centered at $x = 0$.

3. Use the information given in the table below to write the first four nonzero terms of the Taylor series of each function.

	a	$f(a)$	$f'(a)$	$f''(a)$	$f'''(a)$
$f(x)$	5	$-\frac{1}{9}$	$\frac{2}{81}$	$-\frac{8}{729}$	$\frac{16}{2,187}$
$g(x)$	1	3	14	14	6
$h(x)$	2	11	15	22	12

4. Match each function on the left with the corresponding 3rd-degree Taylor polynomial.

$f(x) = \frac{1}{x+1}$ at $x = 2$ $\qquad$ $e + \frac{e}{2}(x-2) + \frac{e}{8}(x-2)^2 + \frac{e}{48}(x-2)^3$

$f(x) = \sin 2x$ at $x = 0$ $\qquad$ $(x-2) - \frac{1}{2}(x-2)^2 + \frac{1}{3}(x-2)^3$

$f(x) = e^{\frac{x}{2}}$ at $x = 3$ $\qquad$ $\frac{1}{3} - \frac{1}{9}(x-2) + \frac{1}{27}(x-2)^2 - \frac{1}{81}(x-2)^3$

$f(x) = \cos x$ at $x = \frac{\pi}{6}$ $\qquad$ $1 - \frac{1}{2}x^2$

$f(x) = \ln(x-1)$ at $x = 2$ $\qquad$ $\frac{\sqrt{3}}{2} - \frac{1}{2}\left(x - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4}\left(x - \frac{\pi}{6}\right)^2 + \frac{1}{12}\left(x - \frac{\pi}{6}\right)^3$

$f(x) = \sin\left(x + \frac{\pi}{2}\right)$ at $x = 0$ $\qquad$ $2x - \frac{4}{3}x^3$

5. True or False? Explain your answer.

a. If the nth derivative of f at $x = 0$ is given by $f^{(n)}(0) = \frac{(-1)^n(n+2)}{(n+1)3^n}$ for $n \geq 1$, then the coefficient for the term containing x^3 in the Maclaurin series is $-\frac{5}{108}$.

b. If the nth derivative of g at $x = 5$ is given by $g^{(n)}(5) = \frac{(-1)^n\ n!}{(n-3)2^n}$ for $n \geq 1$, then the coefficient for the term containing $(x-5)^4$ in the Taylor series is $\frac{1}{16}$.

c. The coefficient for the term containing $(x+1)^5$ in the Taylor polynomial centered at $x = -1$, of $f(x) = x^5 + 2x^3 - x$, is 1.

6. Approximate the following functions around $x = a$ and then approximate $f(a+0.3)$.

$f(x)$	a	$f(a)$	$f'(a)$	$f''(a)$	$P_2(x)$	$f(a+0.3)$
$1 + 5x + 2x^2 - 3x^3$	2					
$\sin\frac{x}{2}$	π					
$\frac{1}{\sqrt{2x+9}}$	0					

10.12 LAGRANGE ERROR BOUND

1. Using the Lagrange error bound, write the appropriate meaning of each term.

$$|R_n| \le \frac{M|x-a|^{n+1}}{(n+1)!}$$

R_n ____

x ____

a ____

n ____

M ____

2. Estimate the given functions using a Taylor polynomial about $x = a$, find the value of M, write the Lagrange error bound, and find the lowest degree polynomial that assures an error smaller than 0.001.

Function	a	M	$\lvert R_n \rvert \le \frac{M\lvert x-a \rvert^{n+1}}{(n+1)!}$	n
$\cos(1.7\pi)$	2π			
$e^{0.90}$	1			
$\sin(-0.2)$	0			

3. Use the Taylor polynomial for each of the following functions about $x=a$ to approximate the value of the function, then find the Lagrange bound for the error, and find the interval $[a,b]$.

a. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$, $x=0$, $\cos(-0.9) =$

b. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$, $x=0$, $e^{-1} =$

c. $f(x) = 5 + 2(x-3) - \frac{4}{2!}(x-3)^2 + \frac{8}{3!}(x-3)^3$, $|f^{(4)}(x)| \le 6$, $f(3.2) =$

d. $2 + \frac{1}{4}(x-4) - \frac{\frac{1}{32}(x-4)^2}{2!}$, $|f^{(3)}(x)| \le \frac{3}{256}$, $x=4$, $f(5) =$

10.13 RADIUS AND INTERVAL OF CONVERGENCE OF POWER SERIES

1. Complete the sequences.

a. A power series is a series of the form $\sum_{n=0}^{\infty} C_n(x-a)^n$, where n is a

______________________.

b. Use the ratio test for a power series $L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$, where $a_n =$

________________.

- if L is infinite, then $R =$ ______.
- if $L = 0$, then $R =$ ______.
- if $L = K|x-a|$, then $R =$ ______.

c. From the ratio test, the series converges when ___________ and diverges when ___________.

d. The interval of convergence is just the interval when ___________ or $a - R < x < a + R$.

2. For the following power series, write a_{n+1}, find the limit using the ratio test, and find the radius of convergence.

Power series	a_{n+1}	$\lim_{n\to\infty} \left\| \frac{a_{n+1}}{a_n} \right\|$	R
$\sum_{n=1}^{\infty} \frac{(-1)^n(x+2)^n}{3^n}$			

$$\sum_{n=1}^{\infty} \frac{(x-5)^{2n}}{(n+1)(-2)^n}$$

$$\sum_{n=0}^{\infty} n^2 x^n$$

$$\sum_{n=0}^{\infty} (n+1)!(x-5)^n$$

$$\sum_{n=1}^{\infty} \frac{(x+3)^n(-2)^n}{(2n+1)}$$

$$\sum_{n=0}^{\infty} \frac{n(x-4)^n}{(3n+1)!}$$

3. True or False? Explain your answer.

a. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n}$ is $4 \le x < 6$.

b. The interval of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{5^n}$ is $-5 \le x < 5$.

c. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x+5)^n}{(n+1)(-5)^n}$ is $-10 < x \le 0$.

10.14 FINDING TAYLOR OR MACLAURIN SERIES OF A FUNCTION

1. Match the function on the left with the corresponding power series.

Function	**Series**
$\cos x$	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$
e^x	$1 + x + x^2 + x^3 + x^4 + \dots$
$\frac{1}{1-x}$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$
$\sin x$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

2. True or False? Explain your answers.

   a. The first three non-zero terms of the Maclaurin series for $f(x) = \frac{\sin(-x^2)}{x^2}$ are $-1 + \frac{x^4}{3!} - \frac{x^8}{5!}$.

   b. The first three non-zero terms of the Maclaurin series for $f(x) = \frac{3}{5-x}$ are $1 + \frac{3}{5}x + \frac{9}{25}x^2$.

   c. The first three non-zero terms of the Maclaurin series for $f(x) = xe^{-x^3}$ are $1 - x^3 + \frac{x^6}{2!}$.

3. Match the following Maclaurin series on the left with the corresponding expression for $f(x)$ on the right.

Maclaurin series	**Function**
$1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\ldots+\frac{x^{2n}}{(2n)!}$	$\frac{1}{1+8x^3}$
$\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n+3}}{(2n+1)!}$	$-x^2\sin(-x)$
$1-8x^3+64x^6+\ldots+(-2x)^{3n}$	$\cos 2x$
$1-2x^2+\frac{2x^4}{3}+\ldots+\frac{(-4)^n x^{2n}}{(2n)!}$	$\frac{1}{2}(e^x+e^{-x})$
$\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$	$x^2\cos x^2$
$\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+2}}{(2n)!}$	e^{-x}

4. Write the function that we need to use to evaluate each Maclaurin series.

	$f(x)$	**Value**
$1-\frac{\pi^2}{2!}+\frac{\pi^4}{4!}+\ldots+\frac{(-1)^n\pi^{2n}}{(2n)!}$		
$1+3+\frac{9}{2!}+\ldots+\frac{3^n}{n!}$		
$2-\frac{2^3}{2!}+\frac{2^5}{5!}+\ldots+\frac{(-1)^n\ x^{2n+1}}{(2n+1)!}$		

$$1+2+2^2+2^3+\ldots+2^n$$

5. Use the Maclaurin series to evaluate each of the following functions.

a. $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$, then $f(\sqrt{\ln 5}) =$ ____________.

b. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$, then $f\left(\sqrt{\frac{\pi}{2}}\right) =$ ____________.

c. $\sum_{n=0}^{\infty} (-1)^n x^{3n}$, then $f(\sqrt[3]{7}) =$ ____________.

10.15 REPRESENTING FUNCTIONS AS POWER SERIES

1. Match each of the following power series of $f(x)$ with the corresponding power series of $f''(x)$ and then evaluate $f''(0)$.

Power series of $f(x)$	**Power series of $f''(x)$**	$f''(0)$
$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}3x^{2n}}{(2n)!}$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{n-1}}{(n-1)!}$	______
$\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)!}$	$\sum_{n=0}^{\infty} \frac{(2n+2)(2n+1)x^{2n}}{n!}$	______
$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{3(2n+1)!}$	$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}3x^{2n-2}}{(2n-2)!}$	______

$\sum_{n=0}^{\infty} \frac{x^{2n+2}}{n!}$ $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n-1}}{3(2n-1)!}$ ______

2. True or False? Explain your answers.

a. $f(x) = \sum_{n=0}^{\infty} \frac{n+1}{3^{n+1}} x^n$, then $\int_0^1 f(x)\ dx = 3$.

b. $f(x) = \sum_{n=0}^{\infty} \frac{n+2}{4^n} x^{n+1}$, then $\int_0^1 f(x)\ dx = \frac{4}{3}$.