3.1 THE CHAIN RULE

1. Chain rule gives us a way to differentiate composite functions, and a composite function is a function of another function.

2. Circled the functions where we'll need to apply chain rule.

$y = \sin x$ [✓ $y = \ln(2x^4 + 3)$] $y = e^x$

$y = 8x$ [✓ $y = \tan^2(3x)$] [✓ $y = (9x - 6)^3$]

3. Complete the table with the "inside" of each function.

Function	**"Inside"**
$\ln(-x^2 + 3x + 2)$	$-x^2 + 3x + 2$
$\sec(\pi x)$	πx
$(2x + 5)^2$	$2x + 5$
e^{7x^4+2}	$7x^4 + 2$

4. What steps do we take to apply chain rule?

   1. Take the derivative of the outside function, leaving the inside function untouched.
   2. Multiply by the derivative of what's inside.

5. Build an "outline" for the derivative of the function, and separately find each value that you need for the outline.

Function

$$f(x)=\frac{e^{x^2}\sin(8x)+(9x-2)\ln(2x^3)}{(7x^2+3x+1)^2}$$

"Outline"

$$f'(x)=\frac{(\quad)(\quad)-(\quad)(\quad)}{(\quad)^2}$$

N = $e^{x^2}\sin(8x)+(9x-2)\ln(2x^3)$

N' = $2xe^{x^2}\sin(8x)+8e^{x^2}\cos(8x)+9\ln(2x^3)+\dfrac{3(9x-2)}{x}$

D = $(7x^2+3x+1)^2$

D' = $2(7x^2+3x+1)(14x+3)$

3.2 IMPLICIT DIFFERENTIATION

1. Whenever we differentiate y, we have to multiply by ____dy/dx____ or by ____y'____.

2. Circle every expression equivalent to the following second derivative:

$$y''=\frac{-(2xy+1)-(e^x-y^2)}{(2xy+1)^2}$$

[✓ $y''=-\dfrac{(2xy+1)+(e^x-y^2)}{(2xy+1)^2}$]

[✓ $y''=-\dfrac{e^x-y^2+2xy+1}{(2xy+1)^2}$]

[✓ $y''=\dfrac{(e^x-y^2)+(2xy+1)}{-(2xy+1)^2}$]

$y''=\dfrac{(e^x-y^2)-(2xy+1)}{-(2xy+1)^2}$

3.3 DIFFERENTIATING INVERSE FUNCTIONS

1. Functions which are inverses of one another are reflections of each other over the line $y = x$.

2. Complete the table by finding $f^{-1}(x)$ for each function.

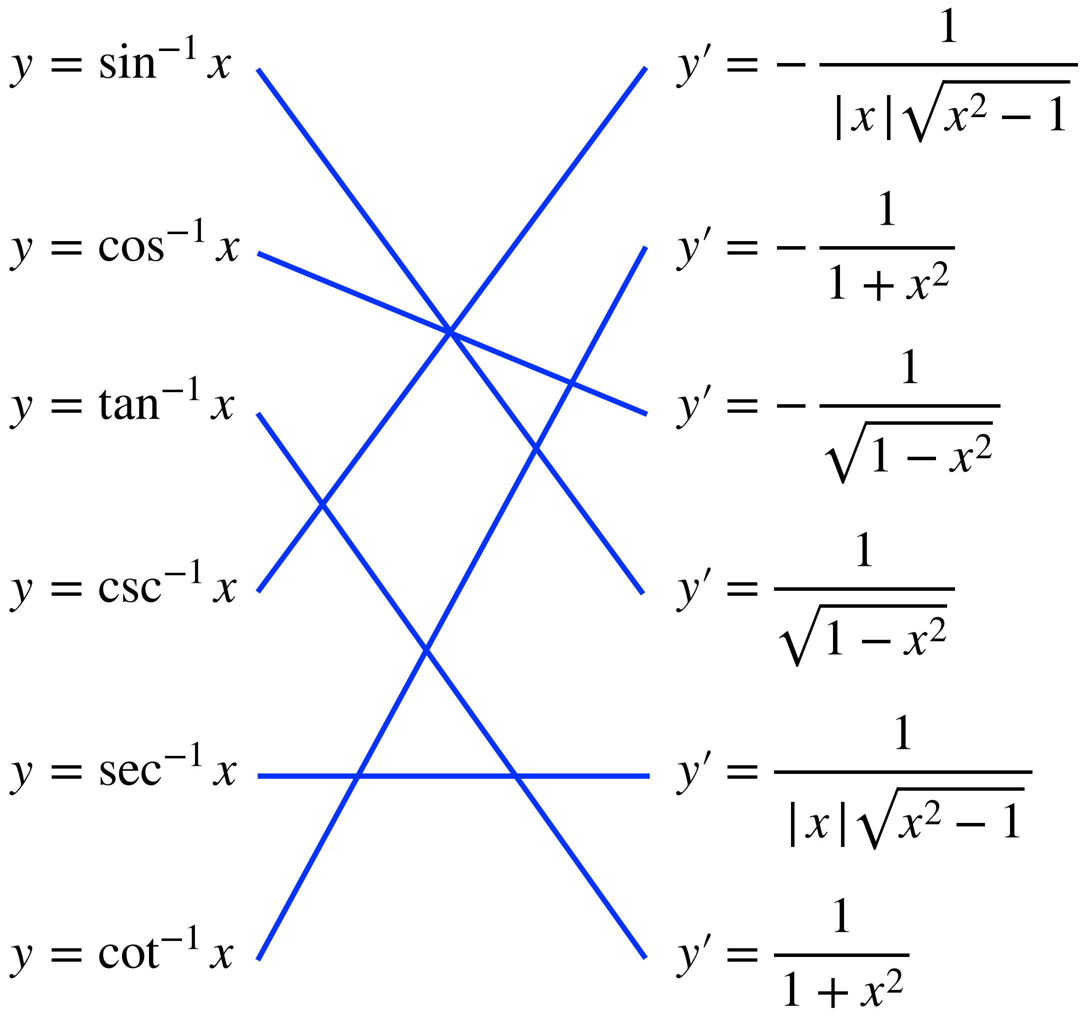
Function	**Inverse function**
$y = x^2 + 1,\ x \geq 0$	$f^{-1}(x) = \sqrt{x - 1}$
$y = \ln x$	$f^{-1}(x) = e^x$
$y = e^{x-6}$	$y = 6 + \ln x$

3.4 DIFFERENTIATING INVERSE TRIGONOMETRIC FUNCTIONS

1. Two things to watch out for:

   1. Use the correct x.
   2. Apply chain rule when the argument isn't x.

2. Match each inverse trig function to its derivative.



$y = \sin^{-1} x$	$y' = -\dfrac{1}{\lvert x\rvert\sqrt{x^2 - 1}}$
$y = \cos^{-1} x$	$y' = -\dfrac{1}{1 + x^2}$
$y = \tan^{-1} x$	$y' = -\dfrac{1}{\sqrt{1 - x^2}}$
$y = \csc^{-1} x$	$y' = \dfrac{1}{\sqrt{1 - x^2}}$
$y = \sec^{-1} x$	$y' = \dfrac{1}{\lvert x\rvert\sqrt{x^2 - 1}}$
$y = \cot^{-1} x$	$y' = \dfrac{1}{1 + x^2}$

3.5 SELECTING PROCEDURES FOR CALCULATING DERIVATIVES

1. Circle the derivative rules that will be used to find the first derivative of each function.

$y = \sin^2(3x)$	[✓ Power]	Product	Quotient	[✓ Chain]
$y = 3e^{x^2}\ln(4x)$	[✓ Power]	[✓ Product]	Quotient	[✓ Chain]
$y = \dfrac{7\sec x}{(9x^2 + 2)e^{4x}}$	[✓ Power]	[✓ Product]	[✓ Quotient]	[✓ Chain]

3.6 CALCULATING HIGHER-ORDER DERIVATIVES

1. The second derivative is just the derivative of the first derivative.

2. In the second derivative of an implicit function, don't forget to substitute for dy/dx.

3. Complete the table so that every value in each row expresses the same function.

y''	$y''(x)$	$\frac{d^2y}{dx^2}$
g''	$g''(x)$	$\frac{d^2g}{dx^2}$
x''	$x''(t)$	$\frac{d^2x}{dt^2}$

4. Find the first and second derivatives of each function.

$f(x)$	$f'(x)$	$f''(x)$
$\sin(3x^2)$	$6x\cos(3x^2)$	$6\cos(3x^2) - 36x^2\sin(3x^2)$
e^{x^2+1}	$2xe^{x^2+1}$	$2e^{x^2+1} + 4x^2e^{x^2+1}$
$\ln(8x-3)$	$\frac{8}{8x-3}$	$-\frac{64}{(8x-3)^2}$