5.1 USING THE MEAN VALUE THEOREM

1. Say whether the Mean Value Theorem applies for each function over the associated interval. If it doesn't apply, explain why not.

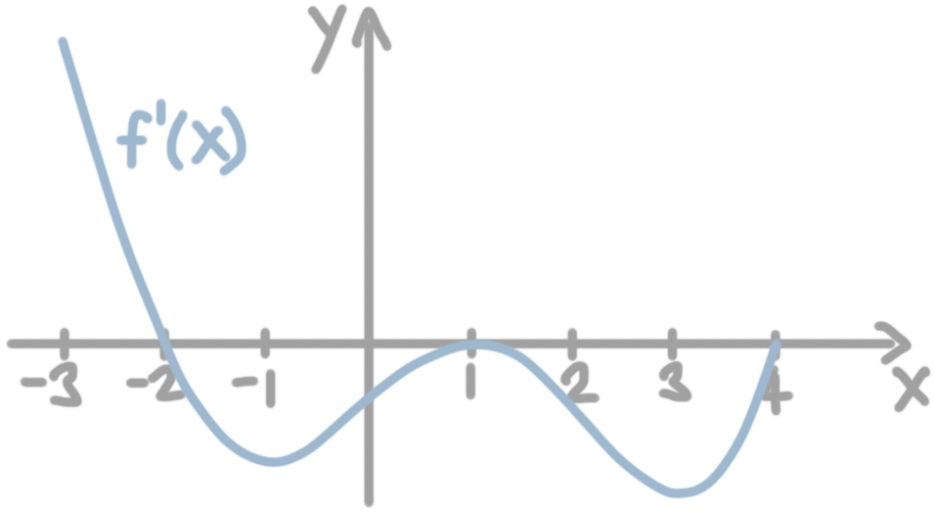
Function	Interval	Does MVT apply?
$f(x) = 5x^2 - 3x + 1$	$[1,3]$	
$g(x) = \dfrac{1}{(x-1)^2}$	$[0,2]$	
$h(x) = \dfrac{x+3}{x-2}$	$[5,10]$	
$m(x) = x^{\frac{2}{3}}$	$[-8,8]$	
$p(x) = x^{\frac{1}{2}}$	$[0,8]$	

2. Find the value $x = c$ over the interval $[1,4]$ where average rate of change on the interval is equal to the instantaneous rate of change of $f(x) = x + \dfrac{4}{x}$.

3. The Mean Value Theorem applies when a function which is _____ on an interval _____ and _____ on _____.

5.2 EXTREME VALUE THEOREM, GLOBAL VERSUS LOCAL EXTREMA, AND CRITICAL POINTS

1. The graph below shows the derivative f' of a function f that's differentiable on the interval $[-3,4]$.



   a. Is f guaranteed to have an absolute maximum and an absolute minimum over the closed interval? Explain.

   b. List all critical points of f in the interval $[-3,4]$.

2. For $f(x) = ax^3 + bx^2 + cx + d$, find the values of a, b, c, and d such that $f(x)$ has critical values at $x = 4$ and $x = -2$, and that $f(0) = 1$ and $f'(0) = 8$.

3. Say whether each of the following statements is true or false.

   a. The slope of a function is 0 at a global extrema.

   _____

   b. The slope of a function is 0 at a local extrema.

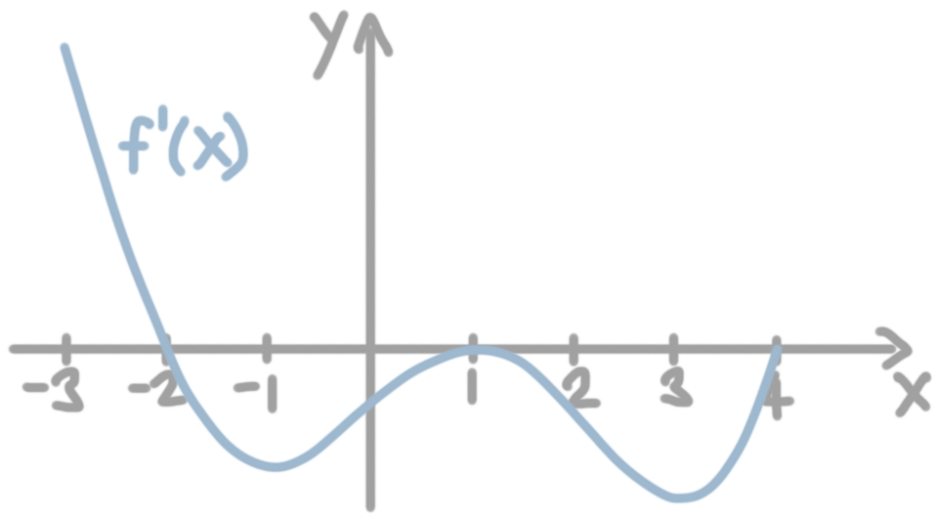
   _____

c. A local extrema can be a global extrema.

d. There can be more than one global maximum.

5.3 DETERMINING INTERVALS ON WHICH A FUNCTION IS INCREASING OR DECREASING

1. The graph below shows the graph of the derivative of a differentiable function f on the interval $[-3,4]$. Using the graph, fill in the table about $f(x)$.



Interval	f' positive or negative?	f increasing or decreasing?
(-3,-2)		
(-2,1)		
(1,4)		

2. For $f(x) = x^2 - 4x + 3$, say where $f(x)$ is increasing and decreasing, and explain how you know.

5.4 USING THE FIRST DERIVATIVE TEST TO DETERMINE RELATIVE (LOCAL) EXTREMA

1. Match each statement with the appropriate justification.

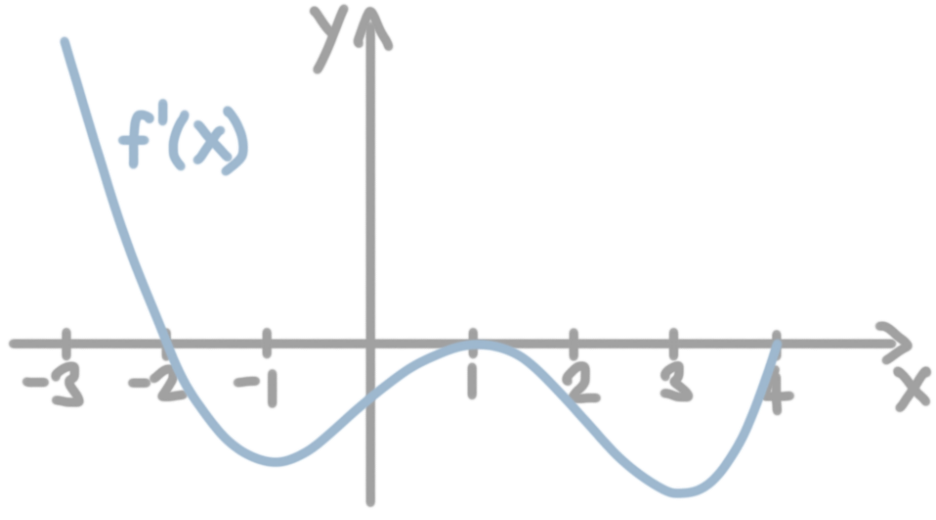
a. f has a relative maximum at $x = a$.

b. f has a relative minimum at $x = a$.

c. f is increasing in the interval (a, b).

d. f is decreasing along the interval (a, b).

2. The graph below shows the derivative of a differentiable function f on the interval $[-3,4]$. Say which statement(s) below are true.



I. $f(x)$ has a relative maximum at $x = 1$ because the graph of $f'(x)$ changes from increasing to decreasing at $x = 1$.

II. $f(x)$ has a relative minimum at $x = -2$ because the graph of $f'(x)$ changes from positive to negative at $x = -2$.

III. $f(x)$ has a relative maximum at $x = -2$ because the graph of $f'(x)$ changes from positive to negative at $x = -2$.

3. The function $f(x) = x^4 - 4x^3$ has two critical values, $x = 0$ and $x = 3$. Fill in each cell of the the table below with "positive" or "negative," then use the table to classify $x = 0$ and $x = 3$ as either a relative maximum, relative minimum, or neither. Justify your answer.

	x<0	0<x<3	x>3
f'(x)			

4. The following table gives information about a differentiable function $g(x)$. Say whether each statement below is true or false.

	-2	0	2	5
g'(x)	3	0	-3	4

a. $g(x)$ has a relative maximum at $x = 0$.

b. $g(x)$ has at least two relative extrema.

c. $x = 0$ is a critical point of $g(x)$.

d. $x = 5$ is a relative maximum of $g(x)$.

5. Fill in the blanks with one of the following terms: positive, negative, increasing, or decreasing.

a. At a relative minimum on $f(x)$, $f(x)$ changes from _____ to _____. At a relative minimum on $f(x)$, $f'(x)$ changes from _____ to _____.

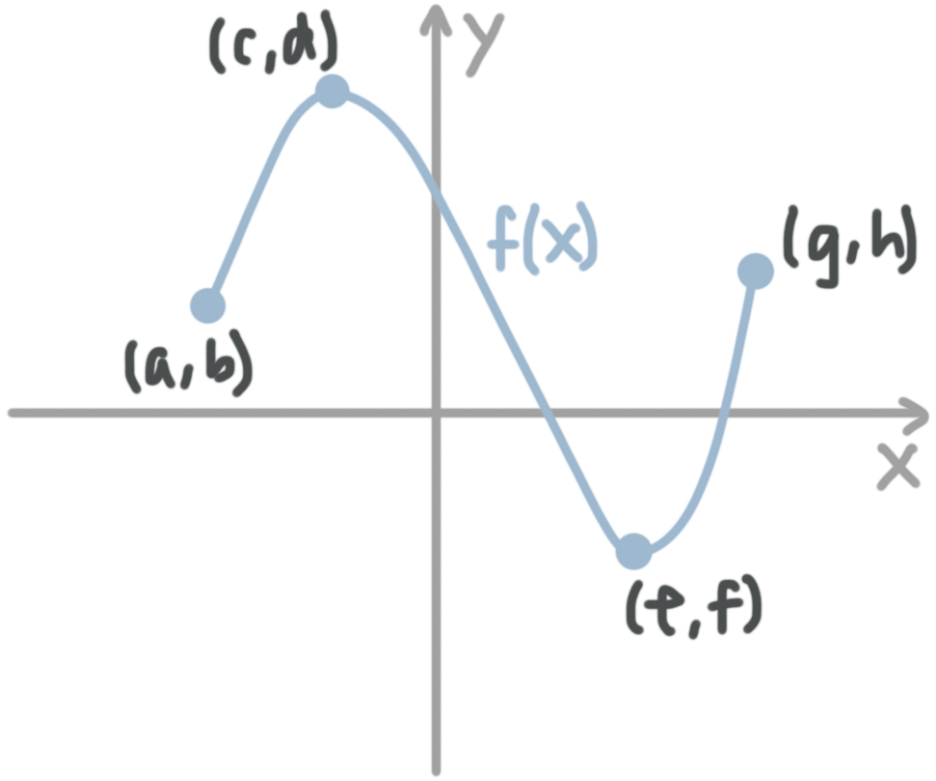
b. At a relative maximum on $f(x)$, $f(x)$ changes from _____ to _____. At a relative maximum on $f(x)$, $f'(x)$ changes from _____ to _____.

5.5 USING THE CANDIDATES TEST TO DETERMINE ABSOLUTE (GLOBAL) EXTREMA

1. For $f(x) = x^3 - 3x^2$ on the interval $[-1,5]$, fill in the table below with all four of the candidates for absolute extrema, and then identify the absolute maximum and absolute minimum values.

x				
f(x)				

2. The graph of $f(x)$ is shown below on the interval $[-3,4]$. Fill in the blanks.



On the given interval, the function has candidates for extrema at:

- _____
- _____

- _____
- _____

In the interval, $f(x)$ has its absolute maximum at _____ and its absolute minimum at _____.

5.6 DETERMINING CONCAVITY OF FUNCTIONS OVER THEIR DOMAINS

1. For $g''(x) = x^2(x-3)(x+4)$, fill in each cell of the table with "positive" or "negative," or "0." Then use the table to answer each question.

	x<-4	-4<x<0	0<x<3	x>3
g"(x)				

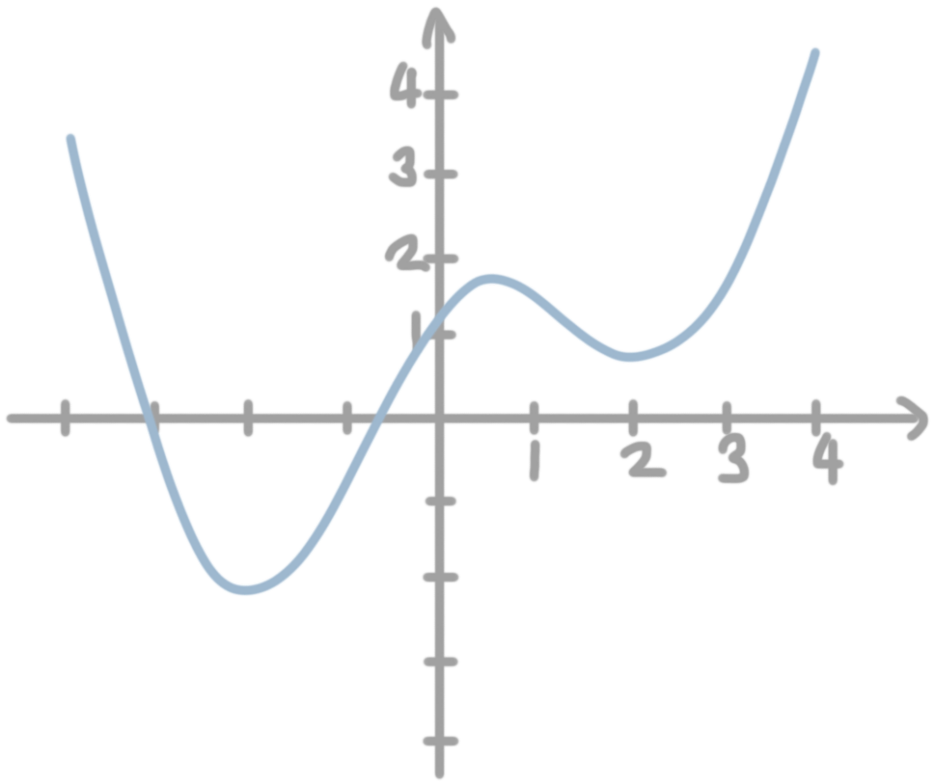
2. For $f'(x) = x^4 - 4x^2$, identify the following:

Intervals where $f(x)$ is concave up:

Intervals where $f(x)$ is concave down:

Number of inflection points:

3. Use the graph of $f(x)$ below to fill in each cell of the table with "concave up" or "concave down."



x	-2	-1	1	2
f(x)				

5.7 USING THE SECOND DERIVATIVE TEST TO DETERMINE EXTREMA

1. Use the information given in the table below to mark each of the following statements as true or false.

x	-3	0	3
f'(x)	0	-2	0
f''(x)	-1	0	5

a. $f(x)$ is concave down at $x = -3$.

b. $f(x)$ is increasing at $x = 0$.

c. $f(x)$ has a relative maximum at $x = 0$.

d. $f(x)$ has a relative maximum at $x = 3$.

e. $f(x)$ has a relative maximum at $x = -3$.

2. For $f(x) = x^4 - 4x^3$, there are two critical values, $x = 0$ and $x = 3$. Complete the following table, then fill in the blanks in the statement below.

x	0	3
f'(0)		
f''(x)		

From the table, we know $x = 0$ is a _____ ("relative maximum," "relative minimum," or "inconclusive") because $f'(0)$ is _____ and $f''(0)$ _____ (<, =, or >) 0. $x = 3$ is a _____ ("relative maximum," "relative minimum," or" inconclusive") because $f'(0)$ is _____ and $f''(0)$ _____ (<, =, or >) 0.

3. Put the following steps in order so that they describe step-by-step how to conduct the second derivative test.

Take the first derivative of $f(x)$

Take the second derivative of $f(x)$

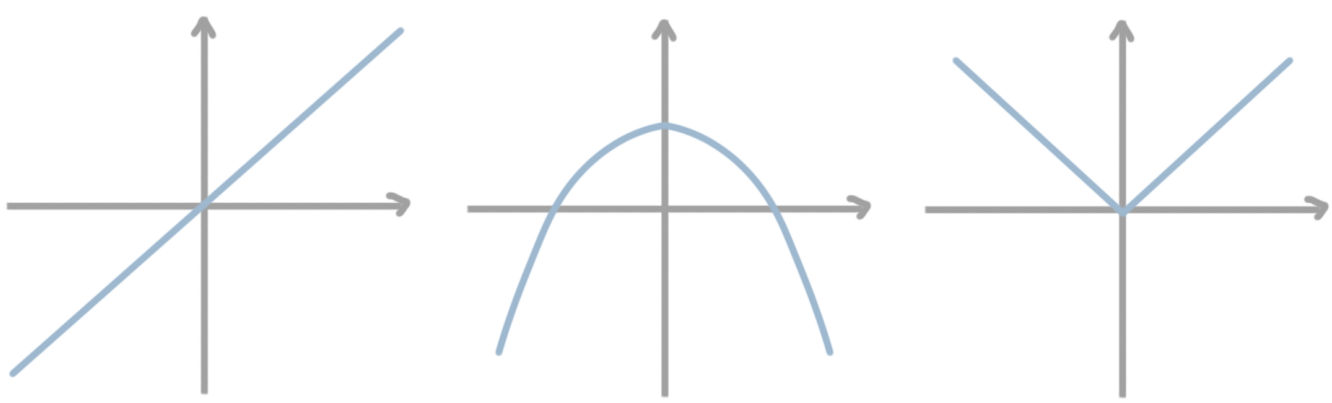
Plug the critical values of into $f''(x)$

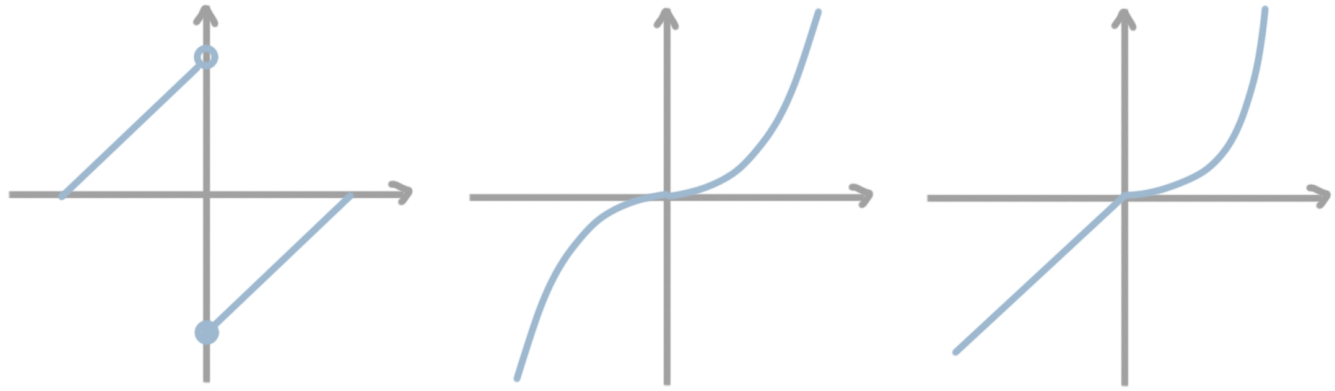
Analyze the sign of the results

Find critical values at which the first derivative is 0 or undefined

5.8 SKETCHING GRAPHS OF FUNCTIONS AND THEIR DERIVATIVES

1. For each graph below, sketch a possible graph of its derivative.



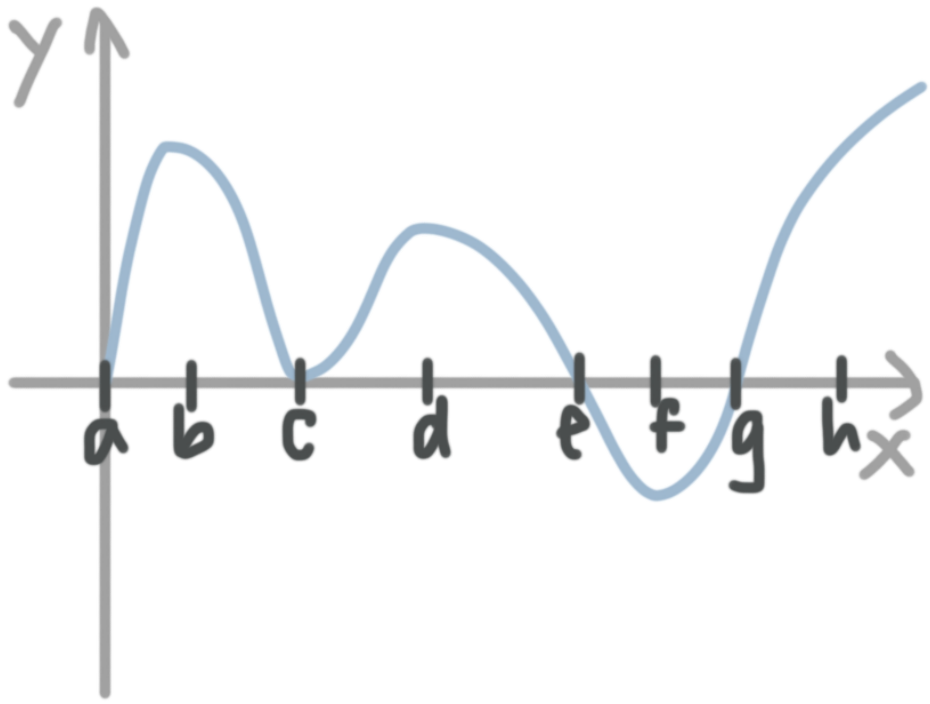


2. Sketch a graph of the continuous function $f(x)$ that meets the criteria below over the interval $[-3,3]$.

$f(0) = 2$ $\quad$ $f''(0) < 0$ $\quad$ $f'(2) > 0$ $\quad$ $f'(-1) > 0$ $\quad$ $f'(0) = 0$

5.9 CONNECTING A FUNCTION, ITS FIRST DERIVATIVE, AND ITS SECOND DERIVATIVE

1. Below is the graph of $g'(x)$ over the closed interval $[a, h]$. Answer the following questions about the graph of $g(x)$, and justify your answers.



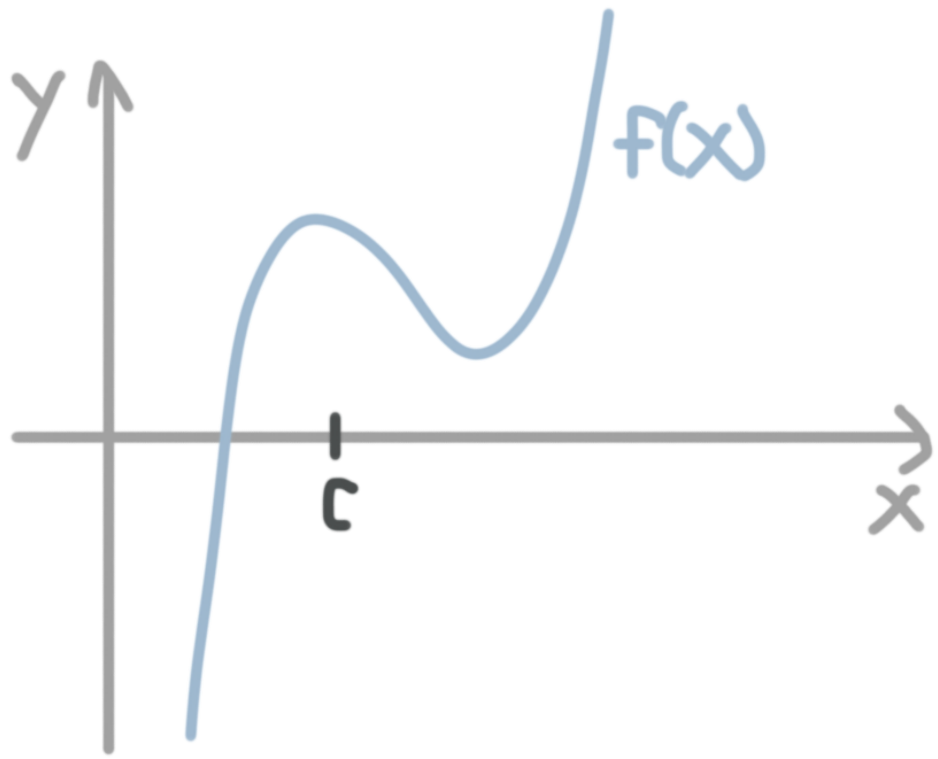
a. List each value of x at which $g(x)$ has a relative maximum.

b. List each value of x at which $g(x)$ has an inflection point.

c. Over which interval(s) is $g(x)$ concave up?

d. Over which interval(s) is $g(x)$ decreasing?

2. The graph of $f(x)$ below is a differentiable function with a horizontal tangent line at $x = c$. Which of the statements below about the relationship between $f(c)$, $f'(c)$, and $f''(c)$ is true?



A $f(c) < f'(c) < f''(c)$

B $f(c) < f''(c) < f'(c)$

C $f'(c) < f''(c) < f(c)$

D $f''(c) < f'(c) < f(c)$

3. Answer each of the questions below.

a. Give two ways to justify the existence of a relative maximum.

b. How would you use the graph of $f''(x)$ to show that $f(x)$ is concave up?

c. How would you use the graph of $f'(x)$ to show that $f(x)$ is concave up?

d. What would you look for on the graph of $f''(x)$ in order to find an inflection point of $f(x)$?

e. What would you look for on the graph of $f'(x)$ in order to find an inflection point on $f(x)$?

5.10 INTRODUCTION TO OPTIMIZATION PROBLEMS

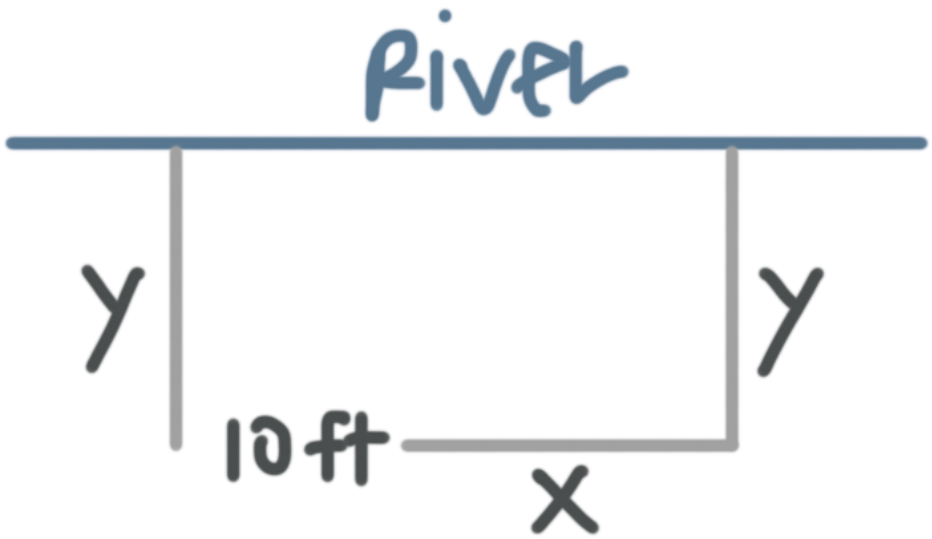
1. For what value of x does $f(x)$ have a relative minimum, given $f'(x) = (x+1)^2(x-3)$?

2. The height of a soccer ball being kicked around on a field can be tracked with by function $h(t) = -t^2 + 25t$, with $h(t)$ measured in feet and t measured in seconds. Find the maximum height of the soccer ball.

5.11 SOLVING OPTIMIZATION PROBLEMS

1. Find two numbers with maximum product that sum to 52.

2. A farmer wants to use 800 feet of fencing to enclose a rectangular cow pasture. He plans to use a straight river as one of the sides of the rectangle and leave a 10-foot wide space for a gate. Find the dimensions of the pasture that will enclose the maximum area by answering each of the questions below.



a. Write down the function the farmer needs to maximize.

b. Write a constraint equation.

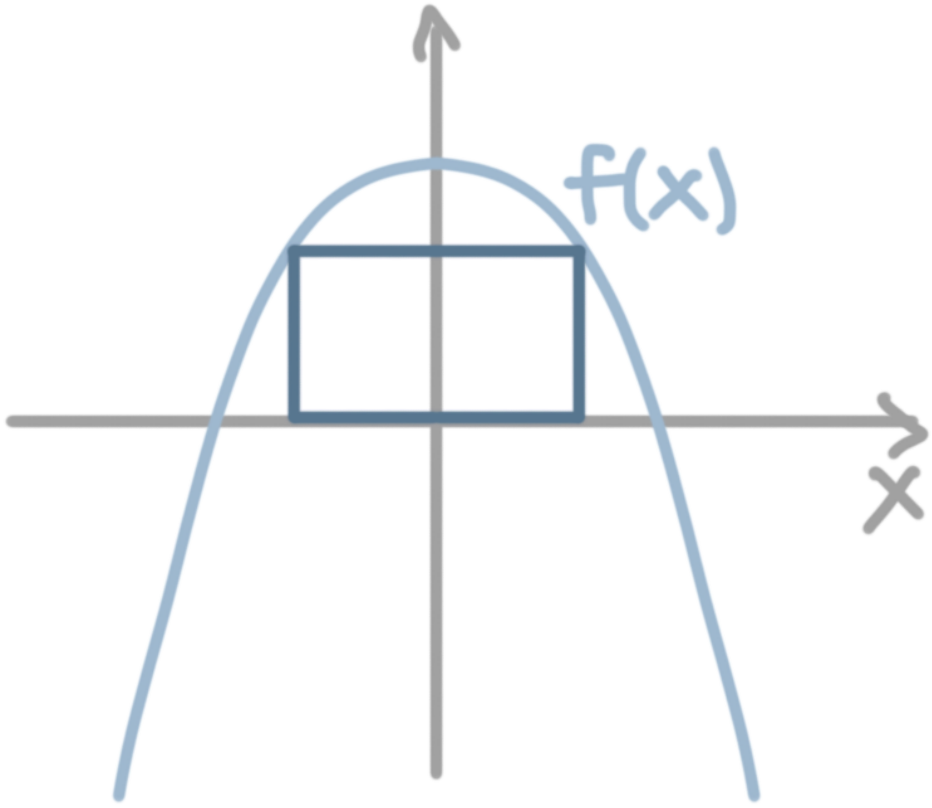
c. Use the constraint equation to write the optimization equation in terms of one variable.

d. Find critical points.

e. What are the values of x and y that maximize the area of the pasture?

f. Find the maximum area of the pasture.

2. The graph shows $f(x) = 9 - x^2$ around an inscribed rectangle. The base of the rectangle sits on the x-axis, with its upper vertices on $f(x)$. Find the maximum area of the rectangle?



5.12 EXPLORING BEHAVIORS OF IMPLICIT RELATIONS

1. The table below relates to some function $y = f(x)$. Fill in the blanks in the statement below with "positive," "negative," "zero," "increasing," "decreasing," or "constant."

(x,y)	(-2,3)	(0,2)	(6,3)
dy/dx	-1/6	0	1/6

At $(-2,3)$, y is _____, since dy/dx is _____. At $(0,2)$, y is _____, since dy/dx is _____. At $(6,3)$, y is _____, since dy/dx is _____.

2. For some $y = f(x)$, read the table below and then say whether each statement below is true or false.

(x,y)	(-1,3)	(0,4)	(3,5)
dy/dx	Undefined	0	2
d^2y/dx^2	-3	4	0

I. At $(-1,3)$, $f(x)$ has a relative maximum.

II. At $(3,5)$, $f(x)$ is concave up.

III. Between $(-1,3)$ and $(0,4)$, $f(x)$ has an inflection point.

IV. At $(0,4)$, $f(x)$ has a vertical tangent line.