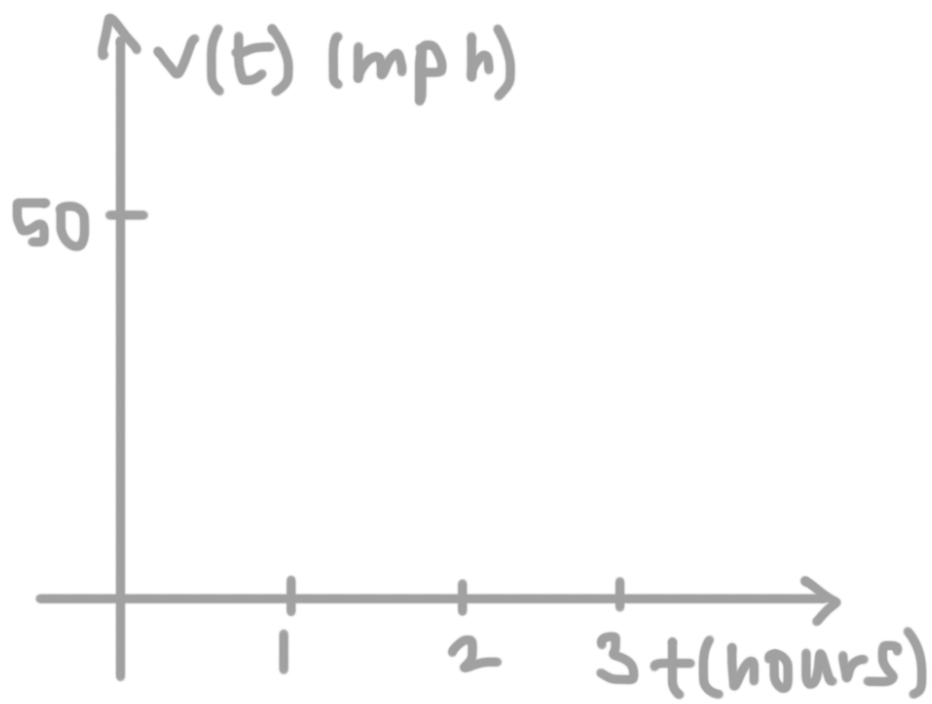
6.1 EXPLORING ACCUMULATIONS OF CHANGE

1. A vehicle travels at 50 miles per hour for 2 hours.

   a. Sketch a graph of the vehicle's velocity, $v(t)$, over time t.



   b. Show with geometry and with calculus how far the vehicle has traveled.

2. Complete the table below with the units for each situation.

	Independent variable	Dependent variable	Units for the area under the curve
Situation 1	hours	miles per hour	
Situation 2	minutes		calories
Situation 3		milliliters per second	milliliters

3. Express each value with a definite integral.

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a. The number of miles run by a runner who runs from $8:00$ a.m. to $10:00$ a.m. at 7 mph.

b. The amount of ice cream being poured out of a machine at 50 gallons per hour over a 4 hour period.

c. The amount of rain collected between $5:00$ a.m. and $11:00$ a.m. when rain collection is modeled by $r(t) = 12 + \cos t$.

6.2 APPROXIMATING AREAS WITH RIEMANN SUMS

1. Using the table below, calculate the left Riemann sum, the right Riemann sum, and the trapezoidal sum for $f(x)$.

x	1	4	6	7
f(x)	9	5	2	10

Left endpoint approximation:

Right endpoint approximation:

Trapezoidal rule approximation:

2. Let $f(x) = x^2 + 1$. Find a midpoint approximation of $\int_0^6 f(x)\,dx$ using 3 rectangles of equal width.

3. Label each statement below as True or False.

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I. A left Riemann Sum will always give a smaller estimation than a right Riemann sum.

II. Computing a right Riemann sum for a function that is increasing will always give an overestimate of the actual area under the curve.

III. A Riemann Sum will always approximate, but never equal, the definite integral over the same interval.

6.3 RIEMANN SUMS, SUMMATION NOTATION, AND DEFINITE INTEGRAL NOTATION

1. The following summations represent Riemann Sums for $f(x) = \frac{9}{x}$ over the interval $[3,8]$ with $n = 30$. Which summation represents left endpoints, which represents right endpoints, and which represents midpoints?

a. $\sum_{k=1}^{30} \frac{9}{3 + \frac{1}{6}(k - 1)} \cdot \frac{1}{6}$

b. $\sum_{k=1}^{30} \frac{9}{3 + \frac{1}{6}(k - 0.5)} \cdot \frac{1}{6}$

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c. $\sum_{k=1}^{30} \frac{9}{3+\frac{1}{6}k} \cdot \frac{1}{6}$

2. Find each value below, then piece them together to convert the definite integral $\int_3^6 x^2\ dx$ into Riemann Sum notation with right endpoints.

$\Delta x =$

$c_k =$

$f(c_k) =$

$$\int_3^6 x^2\ dx = \lim_{x\to\infty} \sum_{k=1}^{n} f(c_k)\Delta x =$$

3. Convert the Riemann Sum notation with right endpoints into definite integral notation.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{\pi}{6n} \sin\left(-\frac{\pi}{6}+\frac{\pi}{6}k\right)\right]$$

$\Delta x = \frac{b-a}{n} =$

$a = c_1 =$

$b - a =$

$b =$

$f(c_k) =$

$f(x) =$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{\pi}{6n} \sin\left(-\frac{\pi}{6}+\frac{\pi}{6}k\right)\right] = \int_a^b f(x)\ dx =$$

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6.4 THE FUNDAMENTAL THEOREM OF CALCULUS AND ACCUMULATION FUNCTIONS

1. For the following integrals, find $\frac{dy}{dx}$.

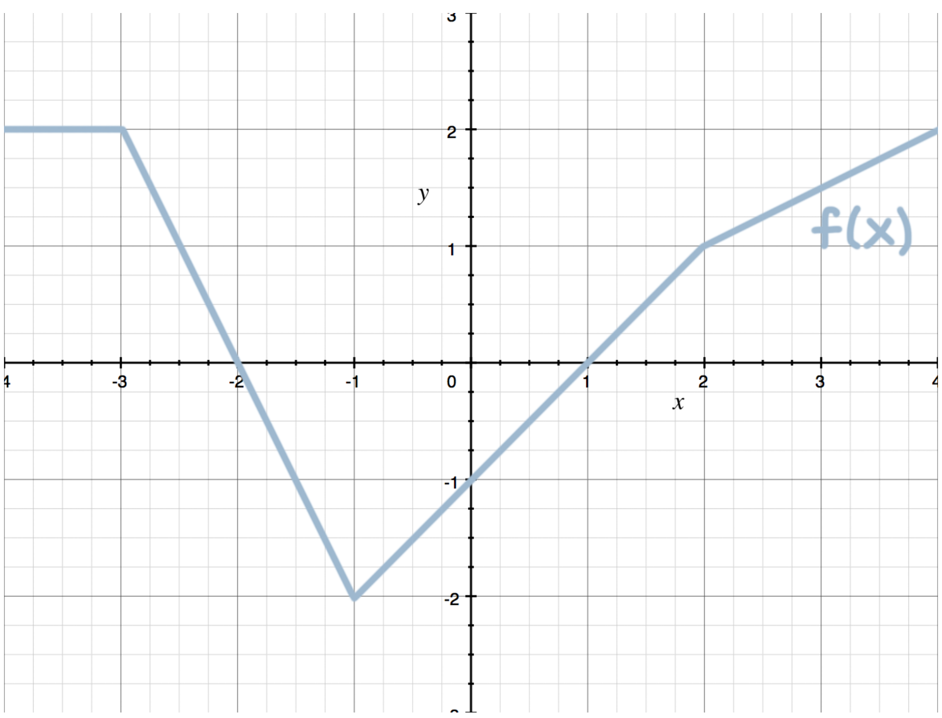
$$y = \int_0^x \sin^2 t \ dt$$

$$y = \int_1^{x^2} e^{t^2} \ dt$$

$$y = \int_2^x (t^3 - t)^5 \ dt$$

$$y = \int_x^6 \ln(1 + t^2) \ dt$$

2. Let $w(x) = \int_1^x f(t) \ dt$. The graph of $f(x)$ is shown below. Find $w(1)$, $w(2)$, $w(-2)$, $w'(2)$, $w''(0)$, and $w''(2)$.



$w(1) =$

$w(2) =$

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$w(-2) =$

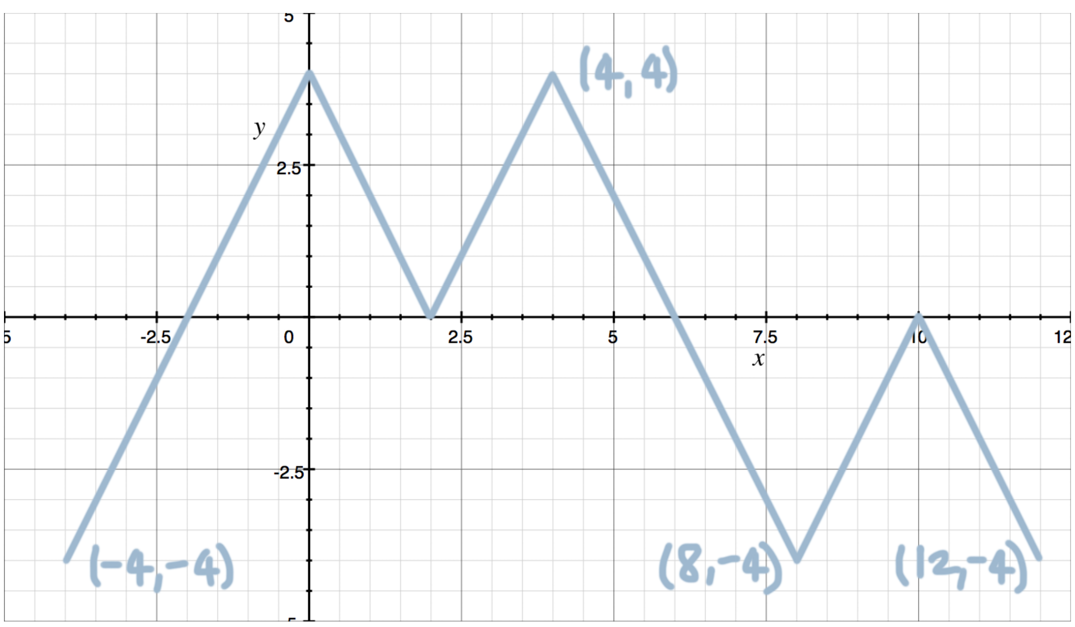
$w'(2) =$

$w''(0) =$

$w''(2) =$

6.5 INTERPRETING THE BEHAVIOR OF ACCUMULATION FUNCTIONS INVOLVING AREA

1. Use the graph of the continuous function f below to evaluate the following integrals.



$\int_{-4}^{0} f(t)\ dt =$

$\int_{-4}^{12} f(t)\ dt =$

$\int_{2}^{-4} f(t)\ dt =$

$\int_{-4}^{12} |f(t)|\ dt =$

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6.6 APPLYING PROPERTIES OF DEFINITE INTEGRALS

1. Given $\int_2^5 f(x)\ dx = -4$ and $\int_8^5 f(x)\ dx = 3$ for a continuous function f, apply properties of definite integrals to find each value.

$\int_5^8 f(x)\ dx =$

$\int_2^2 f(x)\ dx =$

$\int_2^8 f(x)\ dx =$

$\int_2^5 f(x) + 2\ dx =$

2. Calculate $\int_{-3}^3 f(x)\ dx$ for the given function.

$$f(x) = \begin{cases} 4 & x < -2 \\ x^2 & -2 \le x \le 1 \\ x & x > 1 \end{cases}$$

6.7 THE FUNDAMENTAL THEOREM OF CALCULUS AND DEFINITE INTEGRALS

1. Given $g'(x) = x^2 + 4$ and $g(1) = 3$, find each value.

$g(5)$

$g(-1)$

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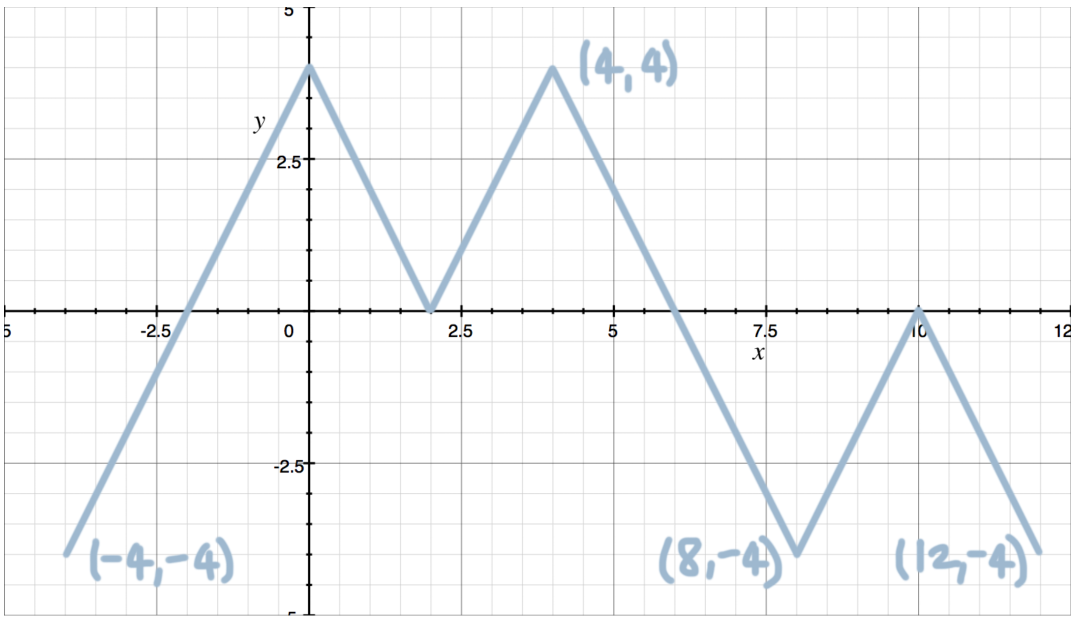
2. Let $h(x) = \int_x^3 f(t)\ dt$ where $f(x)$ is shown in the graph below.



a. Where is $h(x)$ decreasing on the interval $[-4,12]$?

b. Where does $h(x)$ have a relative maximum?

6.8 FINDING ANTIDERIVATIVES AND INDEFINITE INTEGRALS: BASIC RULES AND NOTATION

1. Take the antiderivative of each indefinite integral.

$\int e^x\ dx =$ $\int \sin x + \cos x\ dx =$

$\int \sec^2 x\ dx =$ $\int dx =$

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$$\int 4x - 3\ dx =$$

$$\int \frac{1}{2}x^4 - x^2 + 2\ dx =$$

$$\int \frac{1}{x}\ dx =$$

$$\int x^2(x+1)\ dx =$$

$$\int \sqrt{x} + \frac{1}{\sqrt{x}}\ dx =$$

$$\int \frac{x^2 + 2x}{x}\ dx =$$

2. For $f''(x) = -40x^3 - 6x + 4$, with $f(0) = 2$ and $f'(0) = 1$, find $f(x)$.

6.9 INTEGRATING USING SUBSTITUTION

1. Identify the value you should use for u.

Integral	u
$\int \sin(3x)\ dx$	
$\int x\cos(4x^2)\ dx$	
$\int \sqrt{x+1}\ dx$	
$\int e^{\cos x}\sin x\ dx$	
$\int x^3\sqrt{6+2x^4}\ dx$	

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2. Rewrite the integral $\int_{2}^{4} x\sqrt{x^2+1}\ dx$ using $u = x^2 + 1$.

3. Solve the definite integral $\int_{0}^{4} (2x+1)^{\frac{1}{2}}\ dx$.

4. Take the indefinite integral of $\int \frac{\sin x}{\cos x}\ dx$, using $u = \cos x$.

5. Using the table with information about the twice-differentiable function $f(x)$, evaluate the definite integral $\int_{0}^{4} f''(3x)\ dx$.

x	f(x)	f'(x)	f''(x)
0	5	-1	3
3	2	4	-1
4	4	1	2
12	7	2	1/2

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6.10 INTEGRATING FUNCTIONS USING LONG DIVISION AND COMPLETING THE SQUARE

1. Rewrite each rational integrand using long division, completing the square, reducing the fraction, or factoring. Say which method you used.

$$\int \frac{x^2+5x+6}{x+3}\ dx =$$

$$\int \frac{5x^2+x+1}{x}\ dx =$$

$$\int \frac{3}{x^2+6x+10}\ dx =$$

$$\int \frac{2x^3-17x+7}{x+3}\ dx =$$

2. Below is Finn's work as he integrated the given definite integral. Describe each of Finn's steps.

$$\int_{-2}^{0} \frac{5}{2x^2+4x+4}\ dx$$

$$\frac{5}{2}\int_{-2}^{0} \frac{1}{x^2+2x+2}\ dx$$ Step 1

$$\frac{5}{2}\int_{-2}^{0} \frac{1}{(x^2+2x+1)-1+2}\ dx$$ Step 2

$$\frac{5}{2}\int_{-2}^{0} \frac{1}{(x+1)^2+1}\ dx$$ Step 3

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$u=x+1$ and $\dfrac{du}{dx}=1$, so $du=dx$

$u(0)=-2+1=-1$

$u(-2)=0+1=1$

$\frac{5}{2}\int_{-1}^{1}\frac{1}{u^2+1}\ du$ Step 4

$\frac{5}{2}\arctan u\Big|_{-1}^{1}$ Step 5

$\frac{5}{2}\arctan 1-\frac{5}{2}\arctan(-1)$ Step 6

$\frac{5}{2}\left(\frac{\pi}{4}\right)-\frac{5}{2}\left(-\frac{\pi}{4}\right)=\frac{5\pi}{8}+\frac{5\pi}{8}=\frac{5\pi}{4}$ Step 7

3. Evaluate the indefinite integral.

$$\int\frac{10x^3+13x^2-23x+6}{5x-1}\ dx$$

6.14 SELECTING TECHNIQUES FOR ANTIDIFFERENTIATION

1. Sort the following integrals into the chart below based on the method you'd use to solve each one.

$$\int\frac{\sin(\ln x)}{x}\ dx \qquad \int e^x\sqrt{1+e^x}\ dx$$

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$$\int \frac{x^2+2x}{x-1}\,dx$$

$$\int e^x\,dx$$

$$\int \frac{1}{x^2+4x+13}\,dx$$

$$\int \frac{2x^2+x-3}{x^2}\,dx$$

Integration **Substitution** **Long Division** **CTS**

2. Circle the expressions that are equivalent to $\int x\sqrt{x+2}\,dx$.

$$\int (u-2)u^{\frac{1}{2}}\,du$$

$$\frac{3}{2}\sqrt{u}-\frac{1}{\sqrt{u}}+C$$

$$\frac{2}{15}(x+2)^{\frac{3}{2}}(3x-4)+C$$

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