## 9.1 DEFINING AND DIFFERENTIATING PARAMETRIC EQUATIONS

1. If $x(t) = t^2 + 1$, then $\frac{dx}{dt} =$ ____________. If $y(t) = \frac{2}{3}t^{\frac{3}{2}}$, then $\frac{dy}{dt} =$ ____________.

   This means $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} =$ ____________.

2. A particle moves along a curve in the $xy$-plane with position $((x(t), y(t))$. Complete the following table.

| t | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| dy/dt | 3 | -1 | | 4 |
| dx/dt | 2 | | 5 | |
| dy/dx | | 1/3 | 0 | -8 |

3. Circle the equation of the tangent line for $t = 4$ when $x(t) = 2t + 1$ and $y(t) = \sqrt{t}$ for $t > 0$.

$y - 9 = \frac{1}{8}(x - 2)$

$y - 2 = 8(x - 4)$

$y - 2 = \frac{1}{8}(x - 4)$

$y - 2 = \frac{1}{8}(x - 9)$

## 9.2 SECOND DERIVATIVES OF PARAMETRIC EQUATIONS

1. If $\frac{dx}{dt} = 2$ and $\frac{dy}{dt} = 4t$, find the values below.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} =$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) =$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} =$$

2. The position of a particle moving in the $xy$-plane is given by $x(t) = t^2 - 5$ and $y(t) = t^3$. Find each of the following values.

$$\frac{dx}{dt} =$$

$$\frac{dy}{dt} =$$

$$\frac{dy}{dx} =$$

$$\frac{d^2y}{dx^2} =$$

3. For what values of $t$ over the interval $[0,2\pi)$ is $x = \cos t$ and $y = \sin t$ concave up? Explain your answer.

## 9.3 FINDING ARC LENGTHS OF CURVES GIVEN BY PARAMETRIC EQUATIONS

1. Circle the integral below that will give the length of the curve given by $x(t) = \frac{2}{3}t^3$ and $y(t) = \frac{1}{2}t^2$ from $t = 0$ to $t = 2$.

$$\int_0^1 \sqrt{4t^4 + t^2}\ dt$$

$$\int_0^1 \sqrt{2t^2 + t}\ dt$$

$$\int_0^1 2t^2 + t\ dt$$

$$\int_0^1 \sqrt{\frac{4}{9}t^6 + \frac{1}{4}t^4}\ dt$$

2. Set up and evaluate the integral that will find the length of the curve of $x(t) = \cos t$ and $y(t) = \sin t$ from $t = 0$ to $t = 2\pi$.

## 9.4 DEFINING AND DIFFERENTIATING VECTOR-VALUED FUNCTIONS

1. A particle moves in the plane such that its position at any time $t \geq 0$ is given by $h(t) = \langle 3t^2, 2\cos t \rangle$. Find each of the following values.

$h'(t) =$

$h''(t) =$

$h(0) =$

$h'(0) =$

$h''(0) =$

2. A particle moves in the plane so that its position at any time $t \geq 0$ is given by $h(t) = \langle e^t - t, e^t - e^3 t \rangle$. Choose the statement(s) below that are true about the particle.

   I. At $t = 0$, the particle is at rest.

   II. At $t = 3$, the particle is at rest.

III. The particle is never at rest.

## 9.5 INTEGRATING VECTOR-VALUED FUNCTIONS

1. Fill in the blanks from the answer bank below to form true statements.

$x(t_2) =$ ____________ $+$ ____________

$y(t_1) =$ ____________ $-$ ____________

____________ $= x'(t_1) +$ ____________

$\int_{t_1}^{t_2} y''(t)\, dt =$ ____________ $-$ ____________

Answer bank:

| | | | |
|---|---|---|---|
| $x(t_1)$ | $\int_{t_1}^{t_2} x'(t)\, dt$ | $\int_{t_1}^{t_2} y'(t)\, dt$ | $y'(t_1)$ |
| $x'(t_2)$ | $\int_{t_1}^{t_2} x''(t)\, dt$ | $y'(t_2)$ | $y(t_2)$ |

2. An object moving along a curve in the $xy$-plane has position $(x(t), y(t))$ with $\frac{dx}{dt} = \cos t$ and $\frac{dy}{dt} = \sin t$. At time $t = 0$, the object is at position $(6, -1)$. Where is the particle when $t = \pi$?

3. An object moving along a curve in the $xy$-plane has position $p(t) = \langle x(t), y(t) \rangle$. If the acceleration is given by $a(t) = \langle 2, 4e^{2t} \rangle$ with $p(0) = \langle 2, 1 \rangle$ and the velocity at $t = 0$ is $v(0) = \langle 0, 2 \rangle$, identify the position and velocity vectors for the object.

## 9.6 SOLVING MOTION PROBLEMS USING PARAMETRIC AND VECTOR-VALUED FUNCTIONS

1. A particle moves according to the parametric equations $x(t) = 2t + 4$ and $y(t) = \sqrt{t} - 1$. Fill in the missing values of the table.

| t | 1 | | |
|---|---|---|---|
| x(t) | | 12 | |
| y(t) | | | |
| x'(t) | | | |
| y'(t) | | | 1/6 |

At $t = 1$, choose the statement below that correctly describes the movement of the particle.

I. The particle is moving left and up.

II. The particle is moving left and down.

III. The particle is moving right and up.

IV. The particle is moving right and down.

V. The particle is not moving.

2. A particle's position can be found from $\left(2\cos\left(\frac{t}{2}\right), 4\sin(2t)\right)$ for $t \geq 0$.

Which of the following is the speed of the particle at $t = \pi$?

$0$ $\quad$ $3$ $\quad$ $8$ $\quad$ $\sqrt{65}$

## 9.7 DEFINING POLAR COORDINATES AND DIFFERENTIATING IN POLAR FORM

1. For the polar curve $r = 2\cos\theta$, match each expression on the left to the corresponding value on the right.

| | |
|---|---|
| $x =$ | $2(\cos^2\theta - \sin^2\theta)$ |
| $y =$ | $2\cos^2\theta$ |
| $\frac{dy}{d\theta} =$ | $2\cos\theta\sin\theta$ |
| $\frac{dx}{d\theta} =$ | $-4\cos\theta\sin\theta$ |

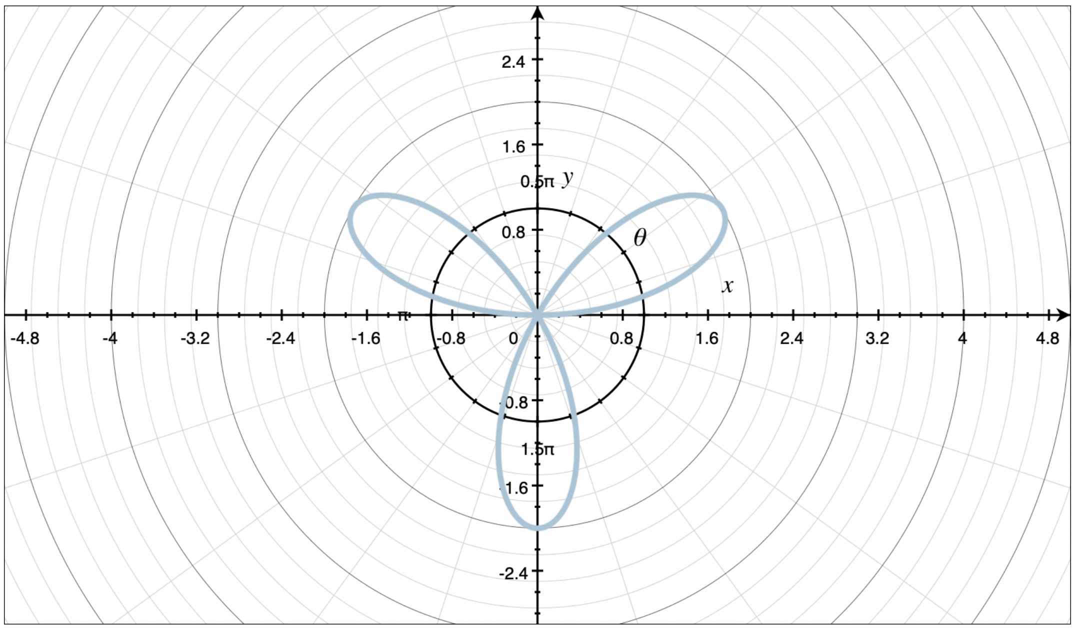
Now use this information to write the equation of the tangent line to the graph at $\theta = \frac{\pi}{4}$.

2. Circle the slope of the line tangent to the polar curve $r = 3\theta^2$ when $\theta = \frac{\pi}{2}$.

$-\frac{4}{\pi}$ $\qquad$ $\frac{\pi}{2}$ $\qquad$ $\frac{3\pi^2}{2}$ $\qquad$ $\frac{\pi}{4}$

## 9.8 FIND THE AREA OF A POLAR REGION OR THE AREA BOUNDED BY A SINGLE POLAR CURVE

1. Below is the graph of $r = 2\sin(3\theta)$. Say which of the following three integrals calculates the total area enclosed by the curve.




I. $2\int_0^{\pi} \sin^2(3\theta)\ d\theta$ $\qquad$ II. $12\int_0^{\frac{\pi}{6}} \sin^2(3\theta)\ d\theta$ $\qquad$ III. $6\int_0^{\frac{\pi}{3}} \sin^2(3\theta)\ d\theta$

2. Which of the following choices give the area of the region enclosed by the polar graph of $r = 3 + 3\sin\theta$.

$\frac{1}{2}\int_0^{\pi} (3 + 3\sin\theta)^2\ d\theta$ $\qquad$ $\int_0^{\pi} (3 + 3\sin\theta)^2\ d\theta$

$$\frac{1}{2}\int_0^{2\pi}(3+3\sin\theta)^2\ d\theta$$

$$\int_0^{\pi}(3+3\sin\theta)^2\ d\theta$$

3. Which of the following choices gives the area of the inner loop of the curve $r=3+6\cos\theta$?

$$\frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}(3+6\cos\theta)^2\ d\theta$$

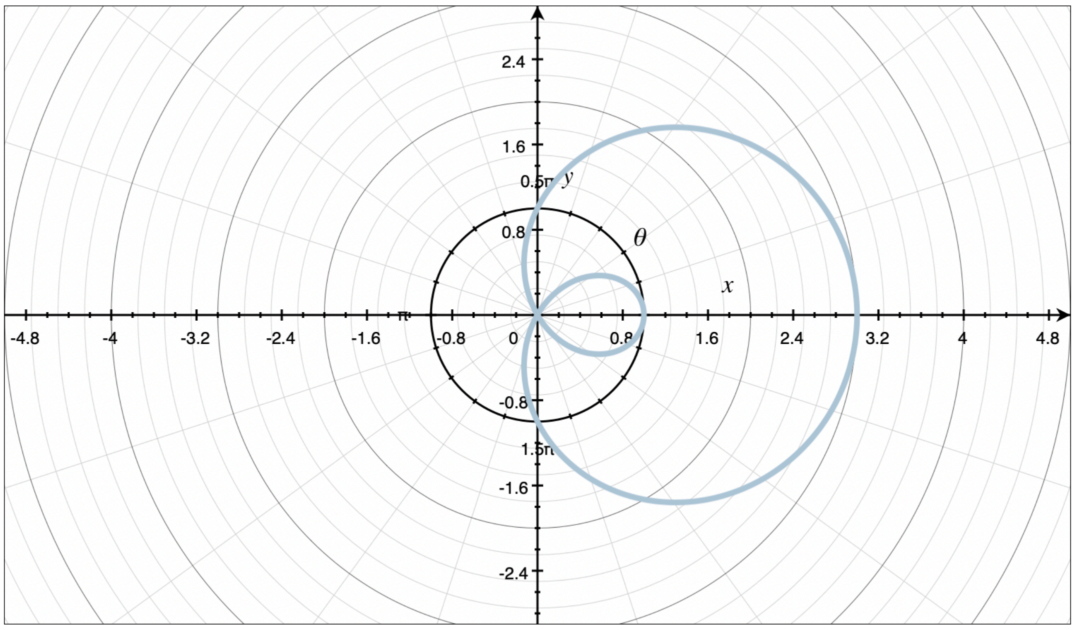
$$\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}(3+6\cos\theta)^2\ d\theta$$

$$\frac{1}{2}\int_0^{2\pi}(3+6\cos\theta)^2\ d\theta$$

$$\frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}(3+6\cos\theta)^2\ d\theta$$

## 9.9 FINDING THE AREA OF THE REGION BOUNDED BY TWO POLAR CURVES

1. Which of the following four choices will not give the area that falls inside $r=2\cos\theta+1$, but outside the smaller loop.



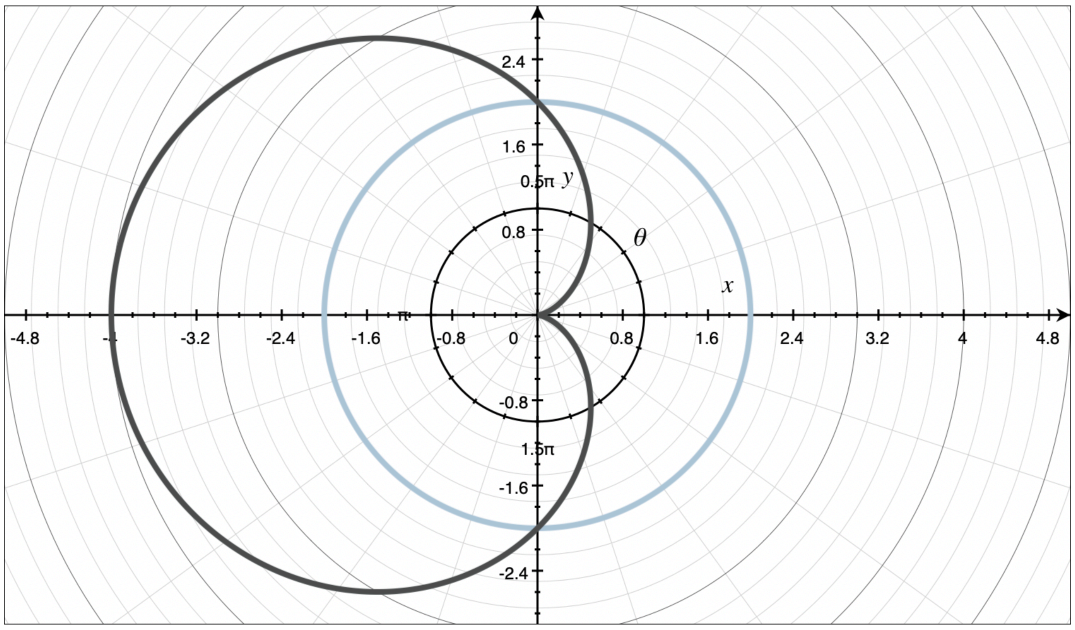
$$\frac{1}{2}\int_{0}^{2\pi}(2\cos\theta+1)^2\ d\theta-\frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}(2\cos\theta+1)^2\ d\theta$$

$$\frac{1}{2}\int_{0}^{\frac{2\pi}{3}}(2\cos\theta+1)^2\ d\theta+\frac{1}{2}\int_{\frac{4\pi}{3}}^{2\pi}(2\cos\theta+1)^2\ d\theta$$

$$\int_{0}^{\frac{2\pi}{3}}(2\cos\theta+1)^2\ d\theta$$

$$\frac{1}{2}\int_{0}^{\frac{4\pi}{3}}(2\cos\theta+1)^2\ d\theta$$

2. The graph below shows $r=2$ and $r=2-2\cos\theta$. Match each of the following integrals with the corresponding statement below.




$$\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}4-(2-2\cos\theta)^2\ d\theta$$

$$\frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(2-2\cos\theta)^2-4\ d\theta$$

$$\frac{1}{2}\int_{0}^{2\pi}4\ d\theta$$

$$\frac{1}{2}\int_{0}^{2\pi}(2-2\cos\theta)^2\ d\theta$$

The area of the region that is:

inside the circle

inside the cardioid

inside the circle and outside the cardioid

inside the cardioid and outside the circle